

Eurocode 2: Design of concrete structures

EN1992-1-1

Symposium Eurocodes: Backgrounds and Applications, Brussels 18-20 February 2008

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22 February 2008

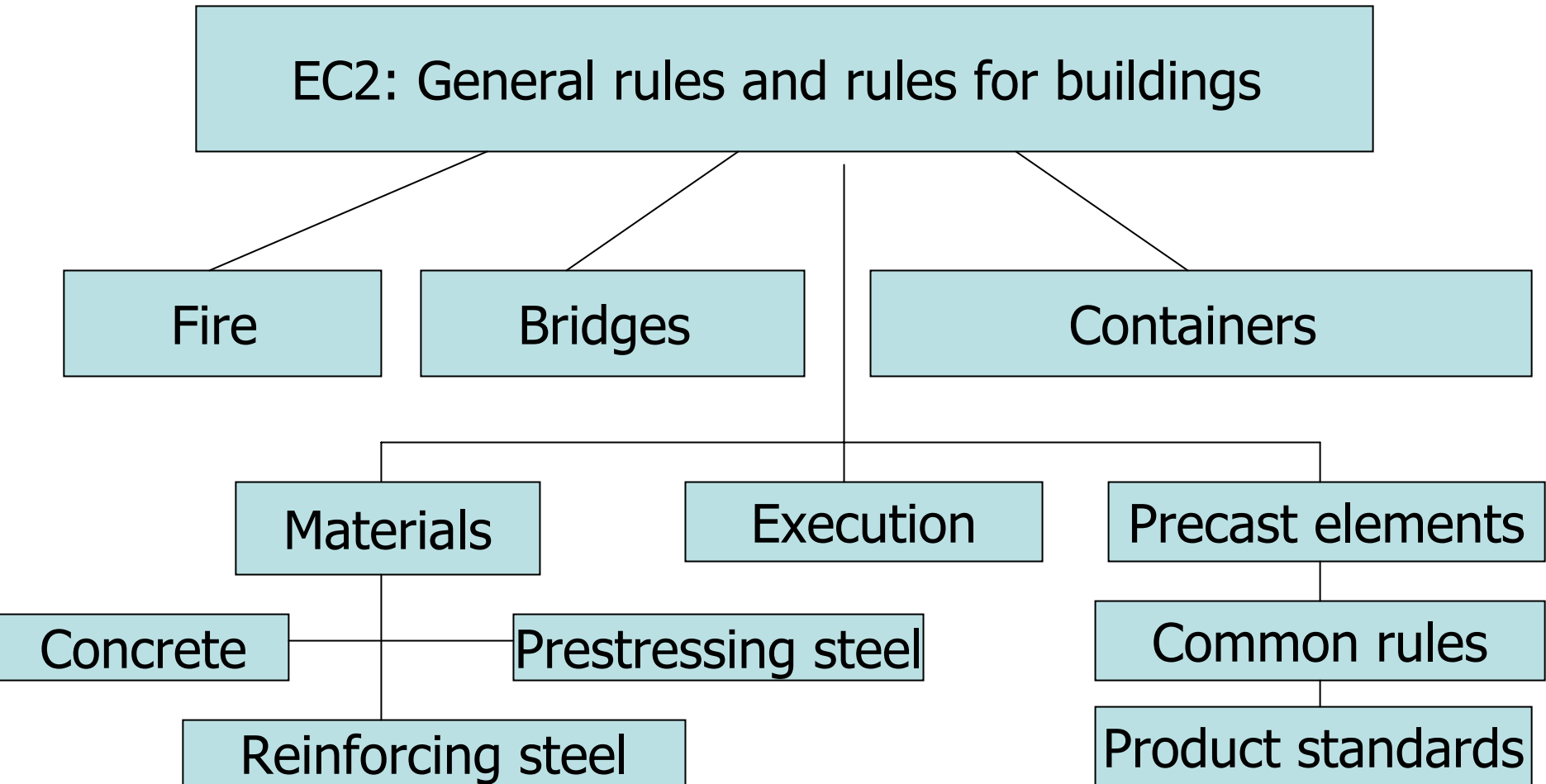
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Requirements to a code

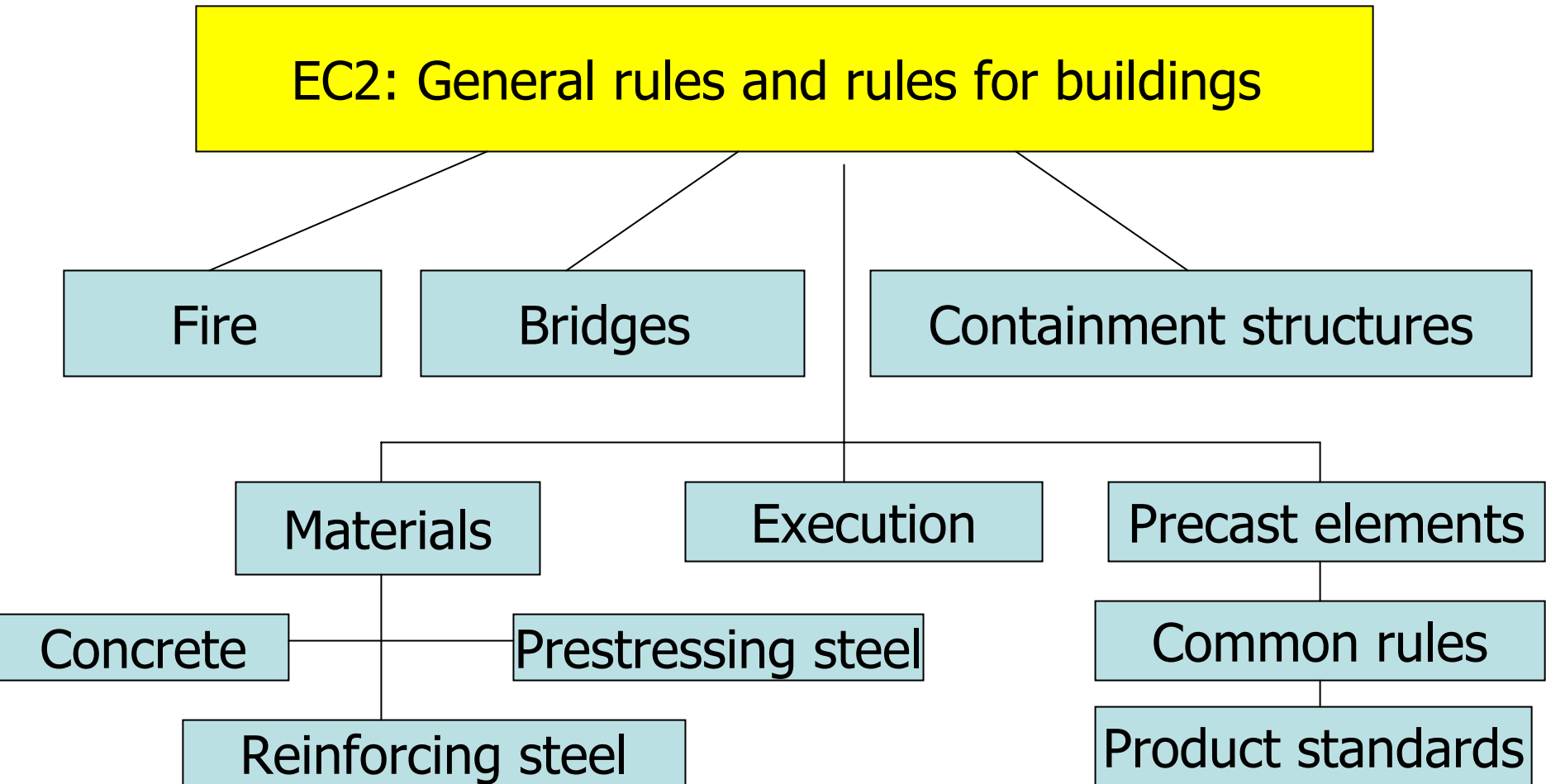
1. Scientifically well founded, consistent and coherent
2. Transparent
3. New developments recognized as much as possible
4. Open minded: models with different degree of complexity allowed
5. As simple as possible, but not simpler
6. In harmony with other codes



EC-2: Concrete Structures



EC-2: Concrete Structures



EN 1992-1-1 "Concrete structures" (1)

Content:

1. General
2. Basics
3. Materials
4. Durability and cover
5. Structural analysis
6. Ultimate limit states
7. Serviceability limit states
8. Detailing of reinforcement
9. Detailing of members and particular rules
10. Additional rules for precast concrete elements and structures
11. Lightweight aggregate concrete structures
12. Plain and lightly reinforced concrete structures



EN 1992-1-1 "Concrete structures" (2)

Annexes:

- A. Modifications of safety factor (I)
- B. Formulas for creep and shrinkage (I)
- C. Properties of reinforcement (N)
- D. Prestressing steel relaxation losses (I)
- E. Indicative strength classes for durability (I)
- F. In-plane stress conditions (I)
- G. Soil structure interaction (I)
- H. Global second order effects in structures (I)
- I. Analysis of flat slabs and shear walls (I)
- J. Detailing rules for particular situations (I)



I = Informative

N = Normative

EN 1992-1-1 “Concrete structures” (3)

In EC-2 “Design of concrete structures –
Part 1: General rules and rules for buildings

109 national choices are possible



Chapter: 3 Materials

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Concrete strength classes

Concrete strength class C8/10 tot C100/115.
(Characteristic cylinder strength / char. cube strength)



A cylinder of high-strength concrete is tested to failure.

**Is concrete
becoming
too strong
to test?**

Concrete strength classes and properties

	Strength classes for concrete													
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0
$f_{ctk,0,05}$ (MPa)	1 1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5
$f_{ctk,0,95}$ (MPa)	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6
E_{cm} (Gpa)	27	29	30	31	32	34	35	36	37	38	39	41	42	44
ϵ_{c1} (‰)	1,8	1,9	2,0	2,1	2,2	2,25	2,3	2,4	2,45	2,5	2,6	2,7	2,8	2,8
ϵ_{cu1} (‰)	3,5									3,2	3,0	2,8	2,8	2,8
ϵ_{c2} (‰)	2,0									2,2	2,3	2,4	2,5	2,6
ϵ_{cu2} (‰)	3,5									3,1	2,9	2,7	2,6	2,6
n	2,0									1,75	1,6	1,45	1,4	1,4
ϵ_{c3} (‰)	1,75									1,8	1,9	2,0	2,2	2,3
ϵ_{cu3} (‰)	3,5									3,1	2,9	2,7	2,6	2,6

Design Strength Values

(3.1.6)

- Design compressive strength, f_{cd}
$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$$
- Design tensile strength, f_{ctd}
$$f_{ctd} = \alpha_{ct} f_{ctk,0.05} / \gamma_c$$

α_{cc} (= 1,0) and α_{ct} (= 1,0) are coefficients to take account of long term effects on the compressive and tensile strengths and of unfavourable effects resulting from the way the load is applied (national choice)

Concrete strength at a time t (3.1.2)

Expressions are given for the estimation of strengths at times other than 28 days for various types of cement

$$f_{cm}(t) = \beta_{cc}(t) f_{cm}$$

where $f_{cm}(t)$ is the mean compressive strength at an age of t days

$$\beta_{cc}(t) = \exp \{s[1-(28/t)^{1/2}]\}$$

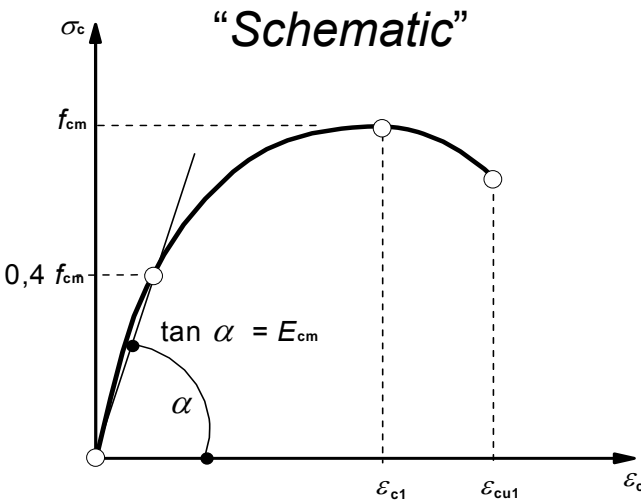
The coefficient s depends on type of cement: $s = 0,20$ for rapid hardening cement (Class R), $s = 0,25$ for normal hardening (Class N) and $s = 0,38$ for Class S (slow hardening) cement. Classes according to EN 197-1

Elastic deformation (3.1.3)

- Values given in EC2 are indicative and vary according to type of aggregate
- $E_{cm}(t) = (f_{cm}(t)/f_{cm})^{0,3} E_{cm}$
- Tangent modulus E_c may be taken as $1,05 E_{cm}$
- Poissons ratio: 0,2 for uncracked concrete
 0 for cracked concrete
- Linear coefficient of expansion $10 \cdot 10^{-6} K^{-1}$

Concrete stress - strain relations (3.1.5 and 3.1.7)

For structural analysis



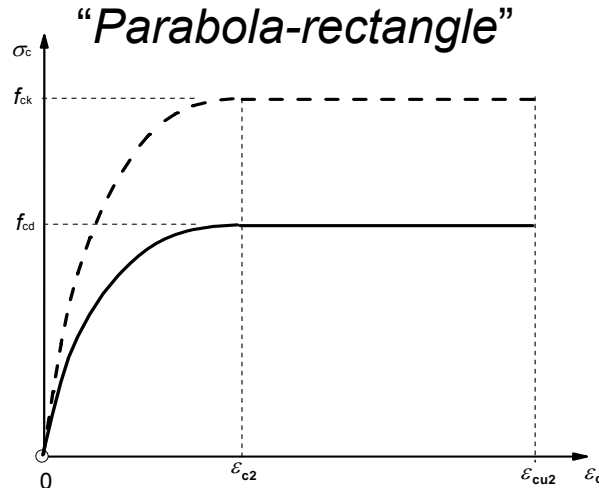
$$\epsilon_{c1} (‰) = 0,7 f_{cm}^{0,31}$$

$$\epsilon_{cu1} (‰) =$$

$$2,8 + 27[(98-f_{cm})/100]^4 f_{cm}/100]^4$$

$$\text{for } f_{ck} \geq 50 \text{ MPa otherwise } 3.5$$

For section analysis



$$\sigma_c = f_{cd} \left[1 - \left(1 - \frac{\epsilon_c}{\epsilon_{c2}} \right)^n \right] \text{ for } 0 \leq \epsilon_c < \epsilon_{c2}$$

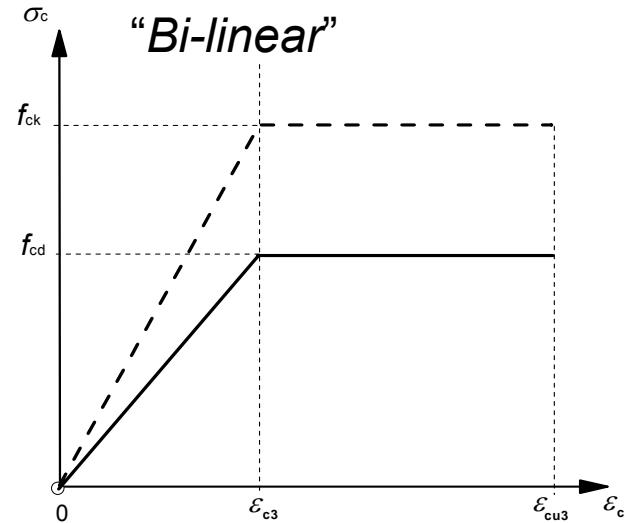
$$\sigma_c = f_{cd} \text{ for } \epsilon_{c2} \leq \epsilon_c \leq \epsilon_{cu2}$$

$$n = 1,4 + 23,4 [(90 - f_{ck})/100]^4$$

$$\text{for } f_{ck} \geq 50 \text{ MPa otherwise } 2,0$$

$$\epsilon_{c2} (‰) = 2,0 + 0,085(f_{ck}-50)^{0,53}$$

$$\text{for } f_{ck} \geq 50 \text{ MPa otherwise } 2,0$$



$$\epsilon_{c3} (‰) = 1,75 + 0,55 [(f_{ck}-50)/40]$$

$$\text{for } f_{ck} \geq 50 \text{ MPa otherwise } 1,75$$

$$\epsilon_{cu3} (‰) = 2,6 + 35[(90-f_{ck})/100]^4$$

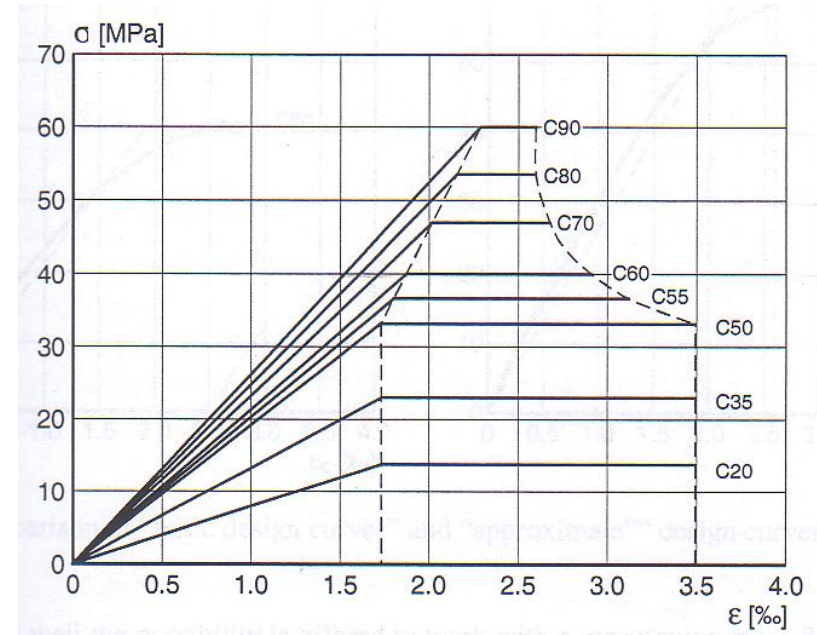
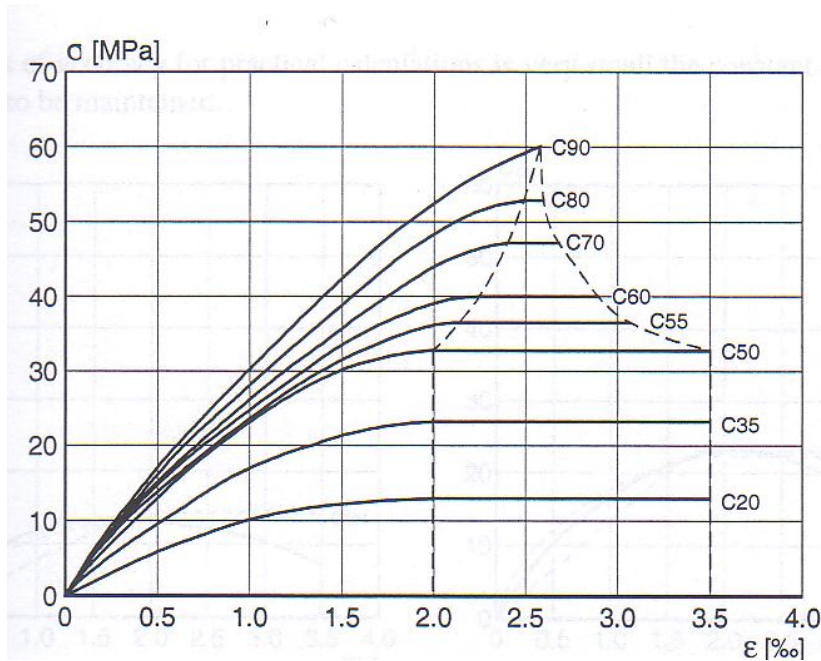
$$\text{for } f_{ck} \geq 50 \text{ MPa otherwise } 3,5$$

$$\epsilon_{cu2} (‰) = 2,6 + 35 [(90-f_{ck})/100]^4$$

$$\text{for } f_{ck} \geq 50 \text{ MPa otherwise } 3,5$$

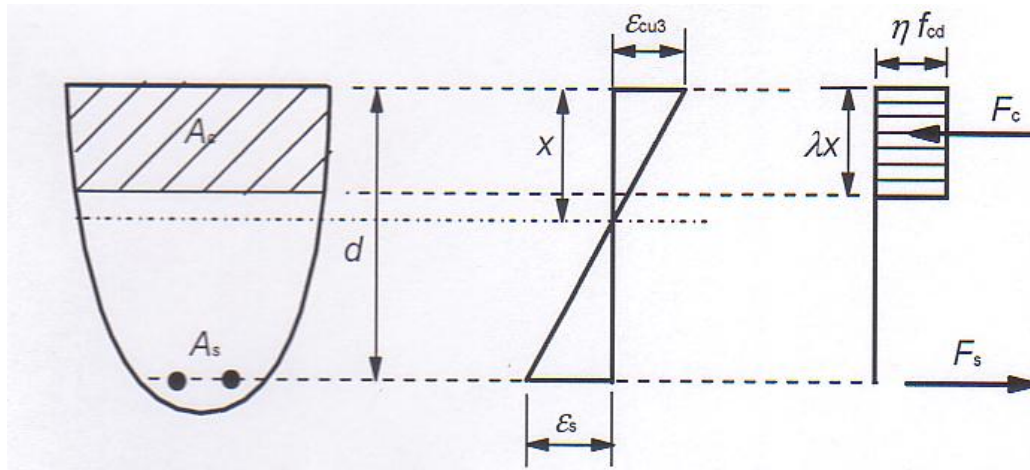
Concrete stress-strain relations

- Higher concrete strength show more brittle behaviour, reflected by shorter horizontal branche



Chapter 3.1: Concrete

Simplified $\sigma - \varepsilon$ relation for cross sections with non rectangular cross-section



$$\lambda = 0,8 \text{ for } f_{ck} \leq 50 \text{ MPa}$$
$$\lambda = 0,8 - (f_{ck} - 50)/400 \text{ for } 50 \leq f_{ck} \leq 90 \text{ MPa}$$

$$\eta = 1,0 \text{ for } f_{ck} \leq 50 \text{ MPa}$$
$$\eta = 1,0 - (f_{ck} - 50)/200 \text{ for } 50 \leq f_{ck} \leq 90 \text{ MPa}$$

Shrinkage (3.1.4)

- The shrinkage strain ε_{cs} is composed of two components:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}$$

where

- drying shrinkage strain

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \quad \text{where } \varepsilon_{cd,0} \text{ is the basic drying shrinkage strain}$$

- autogenous shrinkage strain

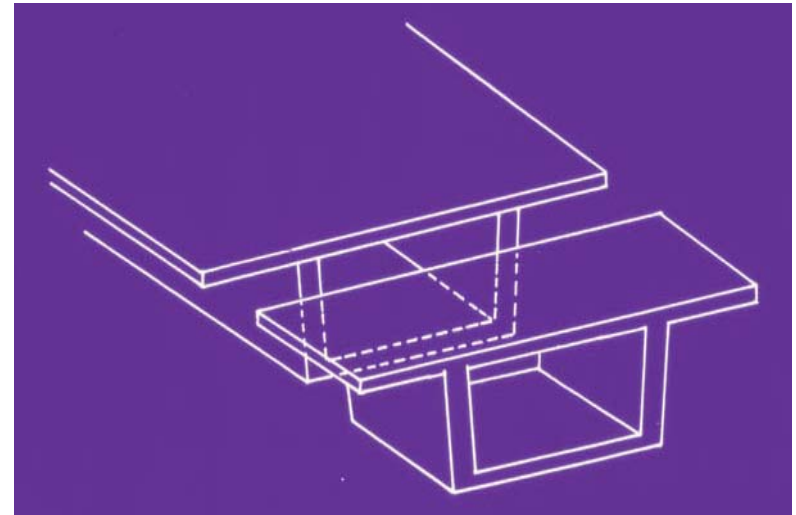
$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

Autogenous shrinkage



Stichtse Bridge, 1997:
Autogenous shrinkage
 $20 \cdot 10^{-3}$ after 2 days

Concrete strength $f_c = 90$ MPa



Shrinkage (3.1.4)

$$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0,04\sqrt{h_0^3}}$$

where t = age of concrete at time considered, t_s = age at beginning of drying shrinkage (mostly end of curing)

$$\varepsilon_{ca}(t) = \beta_{as}(t)\varepsilon_{ca}(\infty)$$

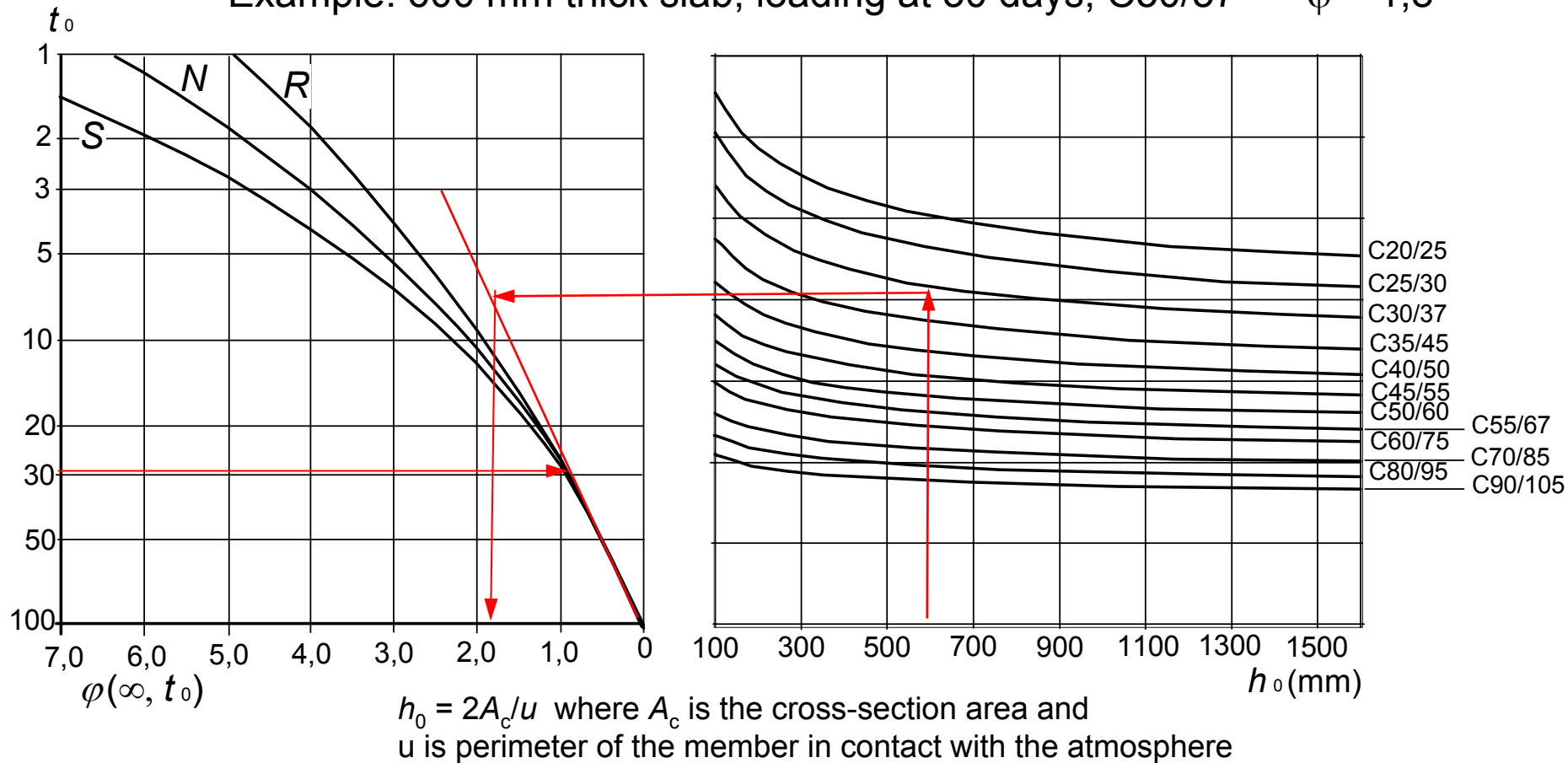
where

$$\varepsilon_{ca}(\infty) = 2,5(f_{ck} - 10) \cdot 10^{-6} \quad \text{and} \quad \beta_{as}(t) = 1 - \exp(-0,2t^{0,5})$$

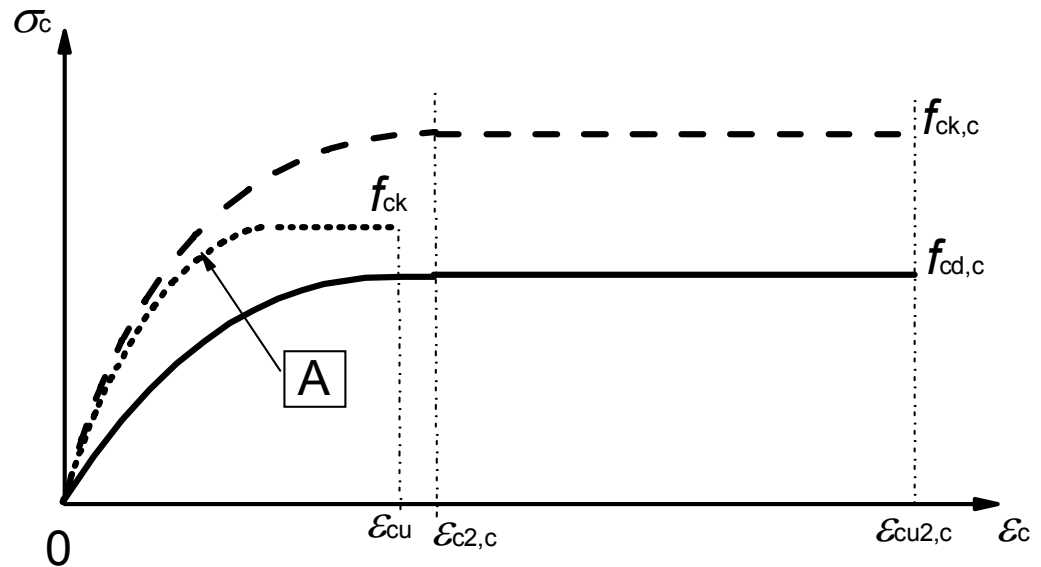
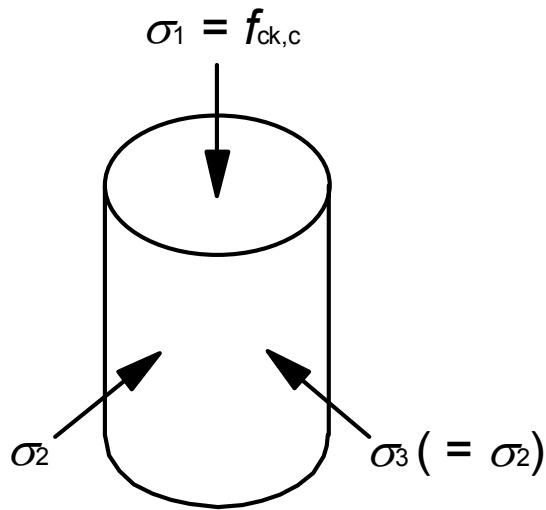
Creep of concrete (3.1.4)

Inside conditions – RH = 50%

Example: 600 mm thick slab, loading at 30 days, C30/37 - $\phi = 1,8$



Confined Concrete (3.1.9)



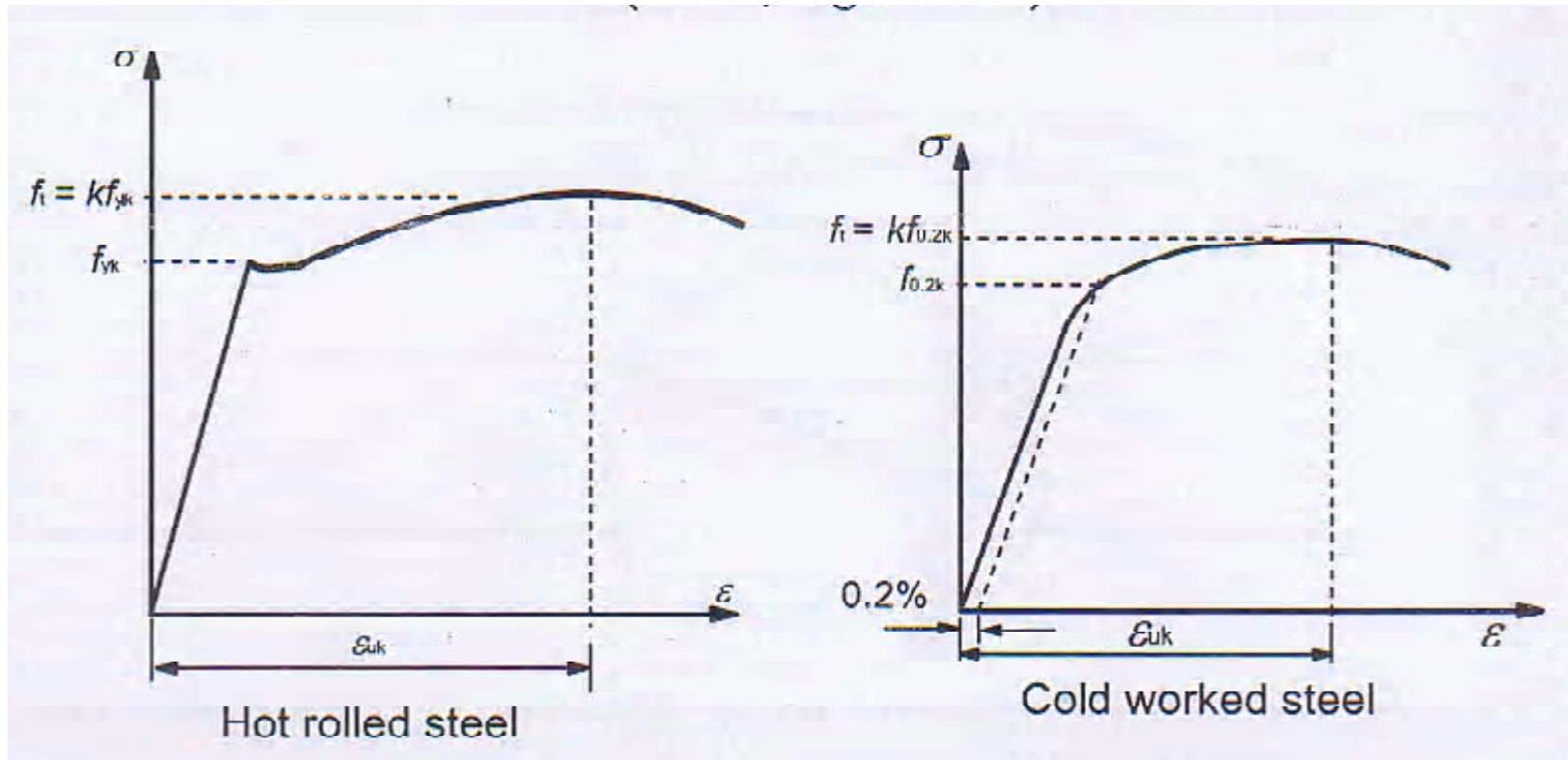
$$f_{ck,c} = f_{ck} (1.000 + 5.0 \sigma_2 / f_{ck}) \quad \text{for } \sigma_2 \leq 0.05 f_{ck}$$

$$= f_{ck} (1.125 + 2.50 \sigma_2 / f_{ck}) \quad \text{for } \sigma_2 > 0.05 f_{ck}$$

$$\varepsilon_{c2,c} = \varepsilon_{c2} (f_{ck,c} / f_{ck})^2$$

$$\varepsilon_{cu2,c} = \varepsilon_{cu2} + 0.2 \sigma_2 / f_{ck}$$

Stress-strain relations for reinforcing steel



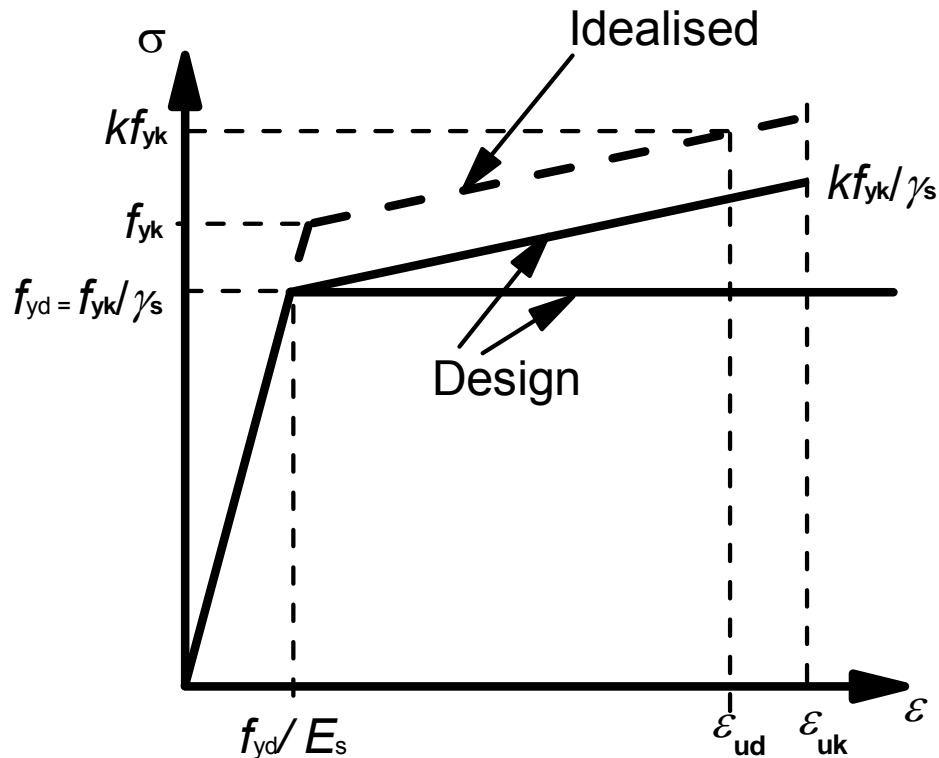
Reinforcement (2) – From Annex C

Product form	Bars and de-coiled rods			Wire Fabrics		
Class	A	B	C	A	B	C
Characteristic yield strength f_{yk} or $f_{0,2k}$ (MPa)	<div> <div>400 to 600</div> <div> <i>cold worked</i> <i>hot rolled</i> <i>seismic</i> </div> </div>					
$k = (f_t/f_y)_k$	$\geq 1,05$	$\geq 1,08$	$\geq 1,15$ $< 1,35$	$\geq 1,05$	$\geq 1,08$	$\geq 1,15$ $< 1,35$
Characteristic strain at maximum force, ε_{uk} (%)	$\geq 2,5$	$\geq 5,0$	$\geq 7,5$	$\geq 2,5$	$\geq 5,0$	$\geq 7,5$
Fatigue stress range (N = 2×10^6) (MPa) with an upper limit of $0.6f_{yk}$	150			100		

Idealized and design stress strain relations for reinforcing steel

Alternative design stress/strain relationships are permitted:

- inclined top branch with a limit to the ultimate strain horizontal
- horizontal top branch with no strain limit



$$k = (f_t/f_y)_k$$

$$\epsilon_{ud} = 0.9 \epsilon_{uk}$$

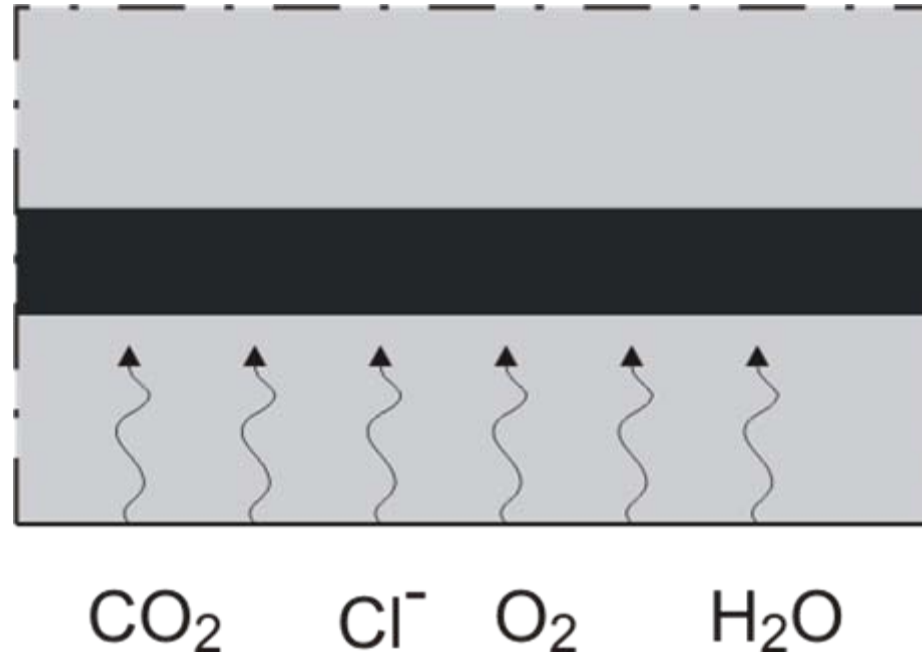
Durability and cover

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22 February 2008

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Penetration of corrosion stimulating components in concrete



Deterioration of concrete

Corrosion of reinforcement by chloride penetration



Deterioration of concrete structures

Corrosion of reinforcement by chloride attack in a marine environment



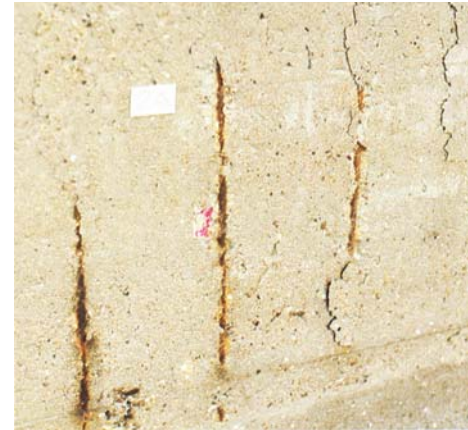
Avoiding corrosion of steel in concrete

Design criteria

- Aggressivity of environment
- Specified service life

Design measures

- Sufficient cover thickness
- Sufficiently low permeability of concrete (in combination with cover thickness)
- Avoiding harmful cracks parallel to reinforcing bars
- Other measures like: stainless steel, cathodic protection, coatings, etc.



Aggressivity of the environment

Main exposure classes:

- The exposure classes are defined in EN206-1. The main classes are:
- XO – no risk of corrosion or attack
- XC – risk of carbonation induced corrosion
- XD – risk of chloride-induced corrosion (other than sea water)
- XS – risk of chloride induced corrosion (sea water)
- XF – risk of freeze thaw attack
- XA – Chemical attack



Agressivity of the environment

Further specification of main exposure classes in subclasses (I)

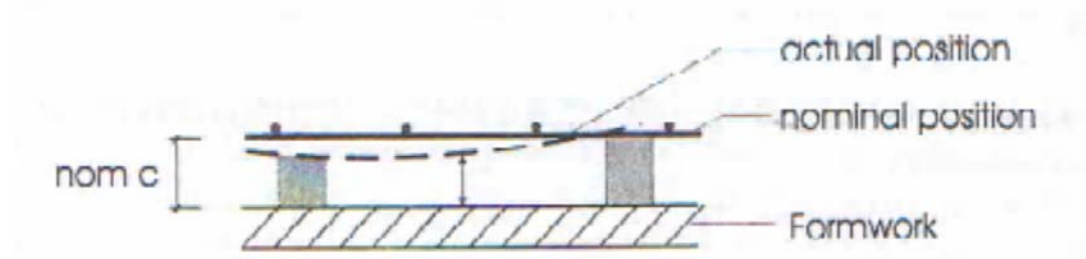
Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
3 Corrosion induced by chlorides		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs

Cover to reinforcement, required to fulfill service life demands

Definition of concrete cover

On drawings the nominal cover should be specified. It is defined as a minimum cover c_{\min} plus an allowance in design for deviation Δc_{dev} , so

$$c_{\text{nom}} = c_{\min} + \Delta c_{\text{dev}}$$



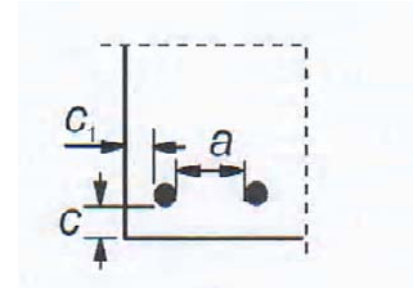
Allowance in design for deviation, Δc_{dev}

The determination of Δc_{dev} is up to the countries to decide, but:

Recommended value 10mm

Reduction allowed if:

- *A quality assurance system is applied including measuring the cover thickness (max. reduction 5mm)*
- *An advanced measuring system is used and non conforming members are rejected (max. reduction 10mm)*



Procedure to determine $c_{\min, \text{dur}}$

EC-2 leaves the choice of $c_{\min, \text{dur}}$ to the countries, but gives the following recommendation:

The value $c_{\min, \text{dur}}$ depends on the “structural class”, which has to be determined first. If the specified service life is 50 years, the structural class is defined as 4. The “structural class” can be modified in case of the following conditions:

- The service life is 100 years instead of 50 years
- The concrete strength is higher than necessary
- Slabs (position of reinforcement not affected by construction process)
- Special quality control measures apply

The finally applying service class can be calculated with Table 4.3N

Table for determining final Structural Class

Structural Class							
Criterion	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1	XD2 / XS1	XD3 / XS2 / XS3
Design Working Life of 100 years	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2
Strength Class ^{1) 2)}	\geq C30/37 reduce class by 1	\geq C30/37 reduce class by 1	\geq C35/45 reduce class by 1	\geq C40/50 reduce class by 1	\geq C40/50 reduce class by 1	\geq C40/50 reduce class by 1	\geq C45/55 reduce class by 1
Member with slab geometry (position of reinforcement not affected by construction process)	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1
Special Quality Control of the concrete production ensured	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1

Final determination of $c_{\min, \text{dur}}$ (1)

The value $c_{\min, \text{dur}}$ is finally determined as a function of the structural class and the exposure class:

Table 4.4N: Values of minimum cover, $c_{\min, \text{dur}}$, requirements with regard to durability for reinforcement steel in accordance with EN 10080.

Environmental Requirement for $c_{\min, \text{dur}}$ (mm)							
Structural Class	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

Special considerations

In case of stainless steel the minimum cover may be reduced.
The value of the reduction is left to the decision of the countries
(0 if no further specification).

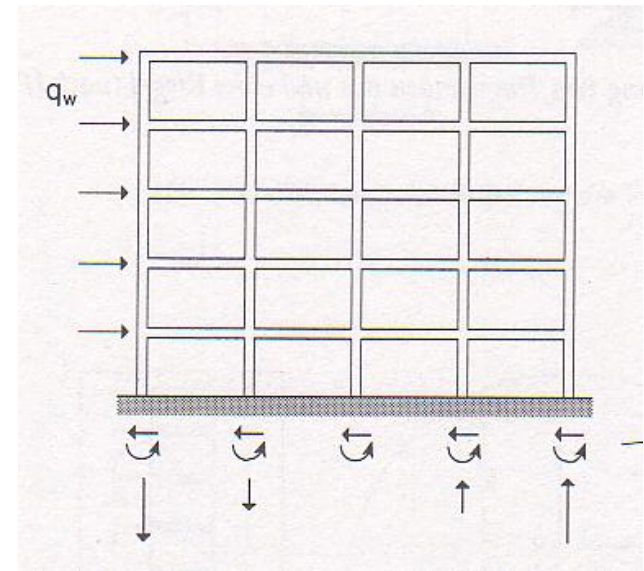


Structural Analysis

Methods to analyse structures

Linear elastic analysis

1. Suitable for ULS and SLS
2. Assumptions:
 - uncracked cross-sections
 - linear $\sigma - \varepsilon$ relations
 - mean E-modulus
3. Effect of imposed deformations in ULS to be calculated with reduced stiffnesses and creep



Geometric Imperfections (5.2)

- Deviations in cross-section dimensions are normally taken into account in the material factors and should not be included in structural analysis

- Imperfections need not be considered for SLS

- Out-of-plumb is represented by an inclination, θ_l

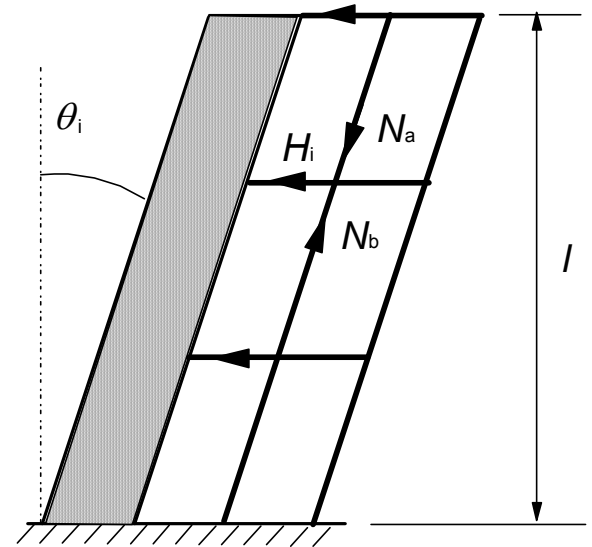
$$\theta_l = \theta_0 \alpha_h \alpha_m \text{ where } \theta_0 = l/200$$

$$\alpha_h = 2/\sqrt{l}; \quad 2/3 \leq \alpha_h \leq 1$$

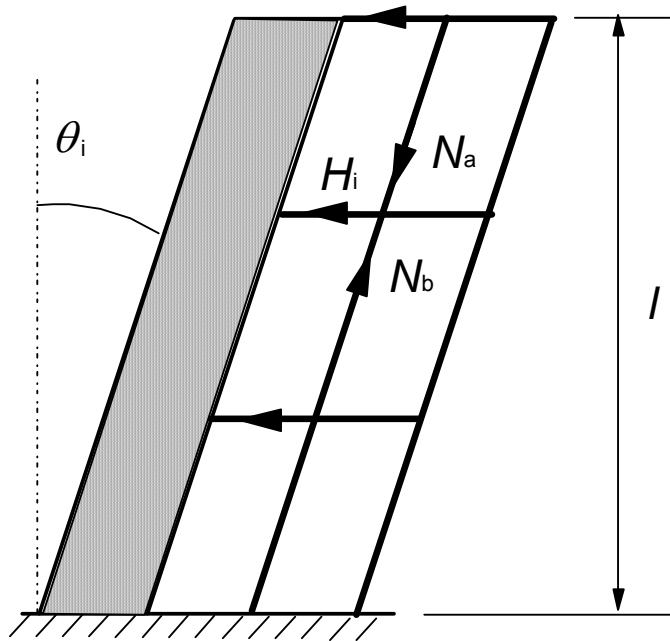
$$\alpha_m = \sqrt{0,5(1+1/m)}$$

l is the height of member (m)

m is the number of vert. members

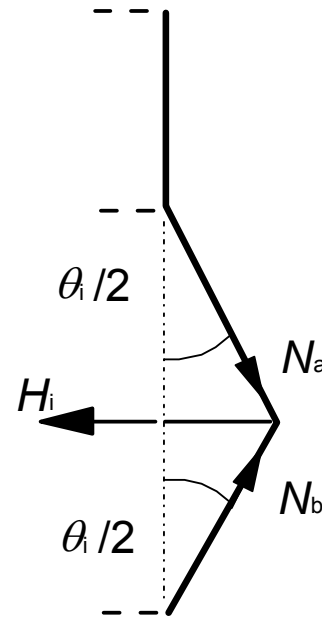


Forces due to geometric imperfections on structures(5.2)



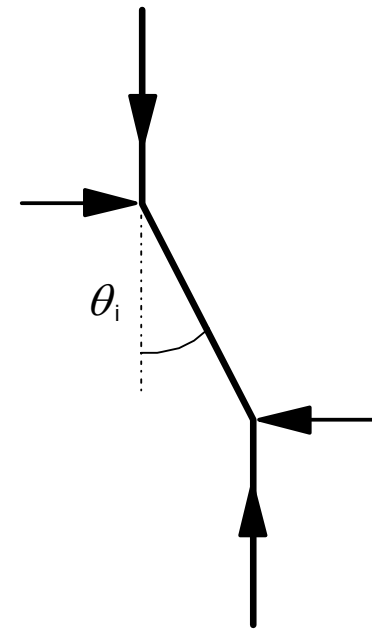
Bracing System

$$H_i = \theta_i (N_b - N_a)$$



Floor Diaphragm

$$H_i = \theta_i (N_b + N_a)/2$$



Roof

$$H_i = \theta_i N_a$$

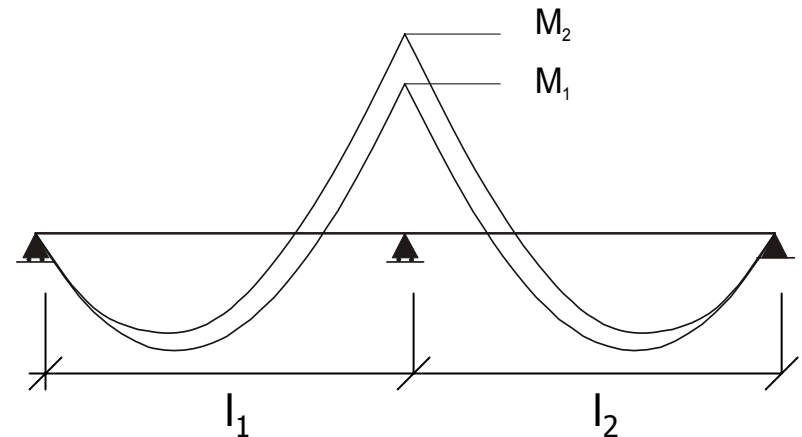
Methods to analyse structures

5.5 Linear elastic analysis with limited redistribution

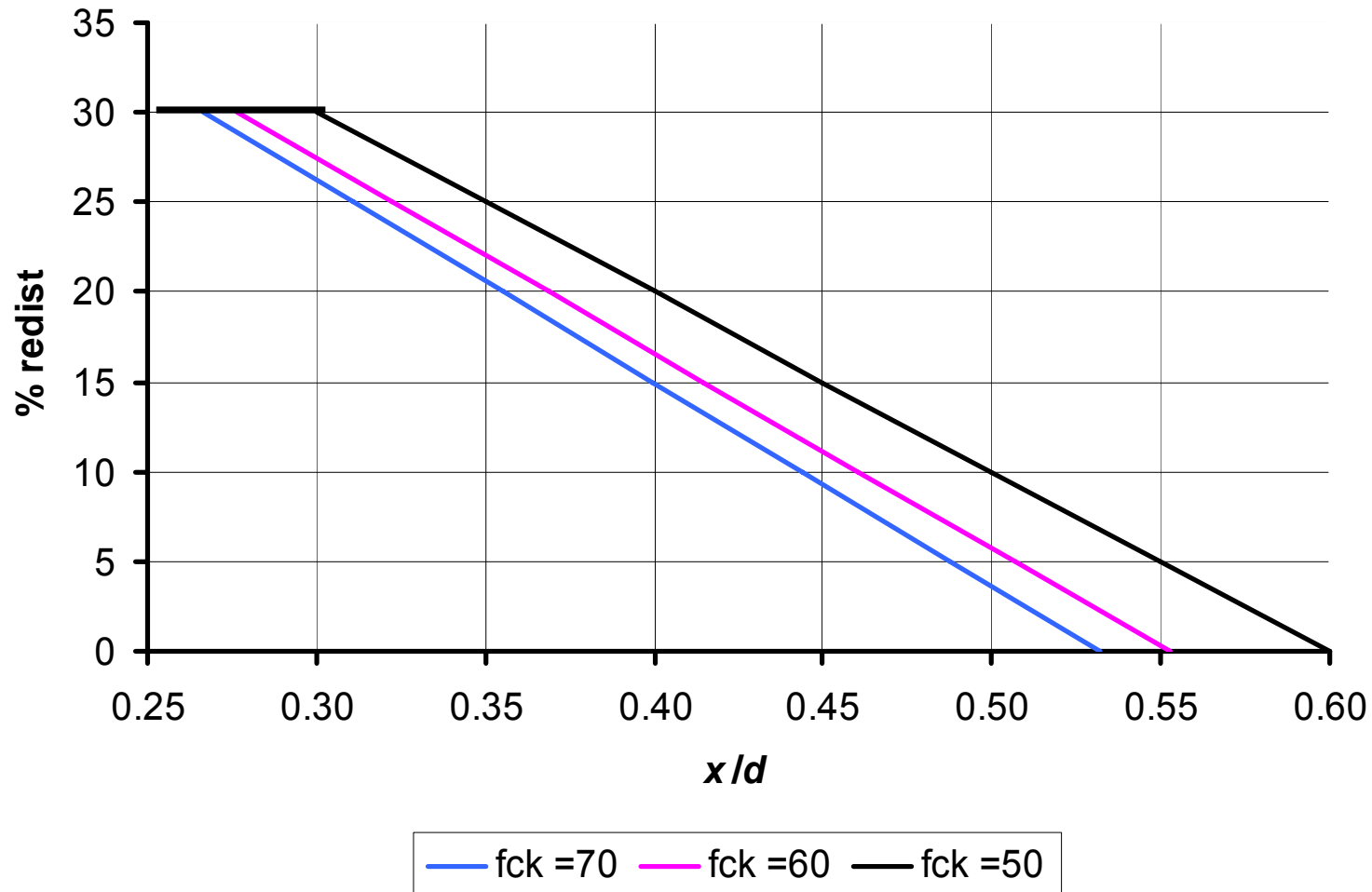
1. Valid for $0,5 \leq l_1 / l_2 \leq 2,0$
2. Ratio of redistribution δ , with
$$\delta \geq k_1 + k_2 x_u / d \text{ for } f_{ck} \leq 50 \text{ MPa}$$
$$\delta \geq k_3 + k_4 x_u / d \text{ for } f_{ck} > 50 \text{ MPa}$$

$\delta \geq k_5$ for reinforcement class B or C

$\delta \geq k_6$ for reinforcement class A



Redistribution limits for Class B & C steel



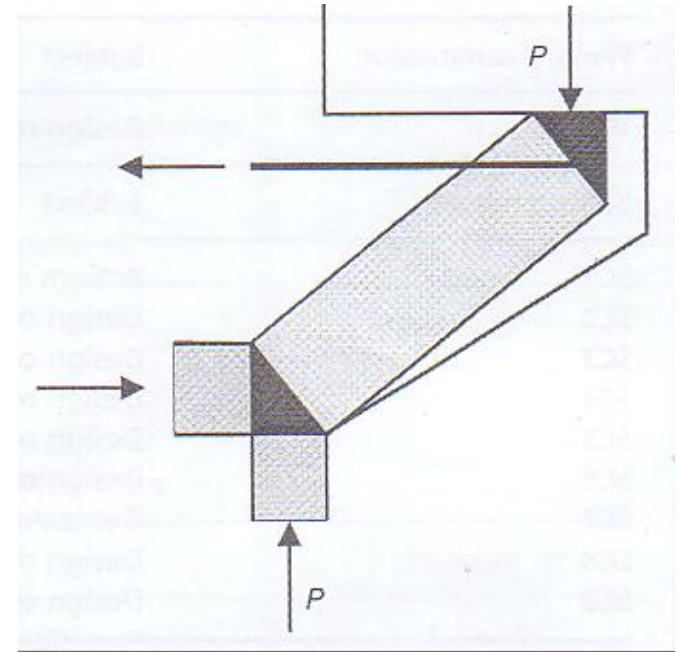
Methods to analyse structures

5.6 Plastic methods of analysis

(a) Yield line analysis

(b) Strut and tie analysis
(lower bound)

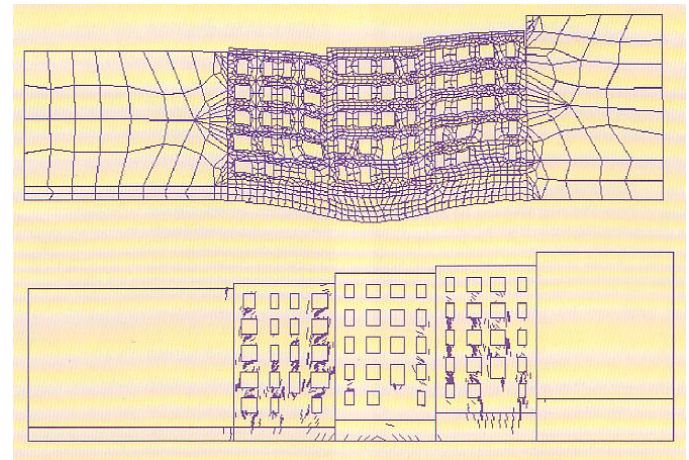
- Suitable for ULS
- Suitable for SLS if compatibility is ensured (direction of struts oriented to compression in elastic analysis)



Methods to analyse structures

Ch. 5.7 Nonlinear analysis

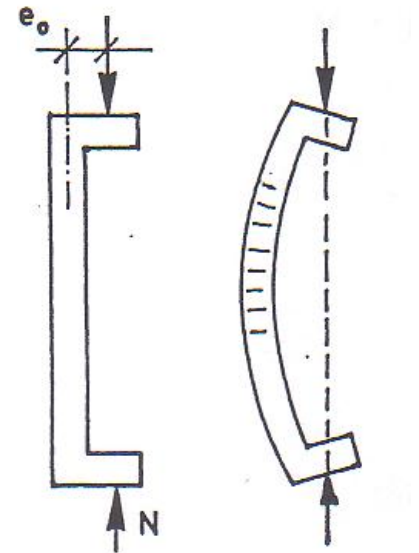
“Nonlinear analysis may be used for both ULS and SLS, provided that equilibrium and compatibility are satisfied and an adequate non-linear behaviour for materials is assumed. The analysis may be first or second order”.



Chapter 5 “Structural analysis”

5.8 Second order effects with axial loads

- Slenderness criteria for isolated members and buildings (when is 2nd order analysis required?)
- Methods of second order analysis
 - General method based on nonlinear behaviour, including geometric nonlinearity
 - Analysis based on nominal stiffness
 - Analysis based on moment magnification factor
 - Analysis based on nominal curvature



Extended calculation tools are given

Methods of analysis

Biaxial bending

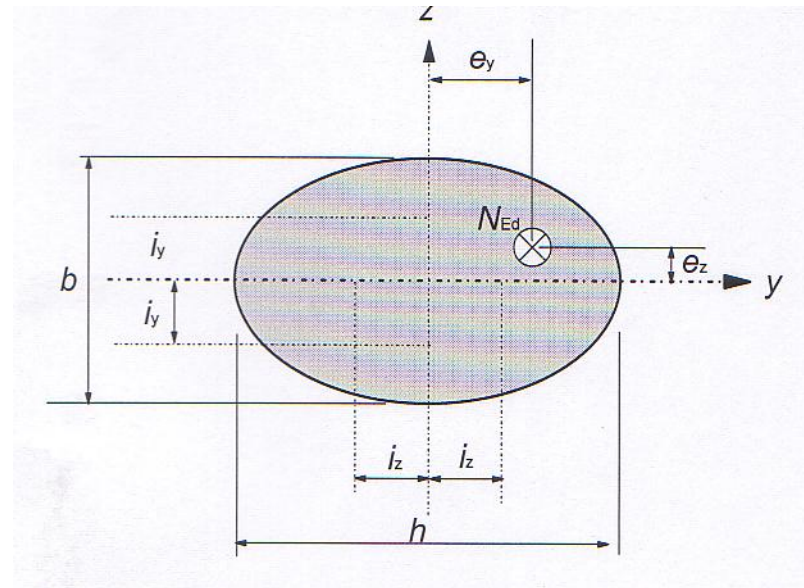
$M_{Rdz/y}$ design moment around
respective axis

$M_{Rdz/y}$ moment resistance in
respective direction

For circular and elliptical cross-section
 $a = 2$.

For rectangular cross section, see table

N_E/N_{Rd}	0,1	0,7	1,0
a	1,0	1,5	2,0



$$\left(\frac{M_{Edz}}{M_{Rdz}} \right)^a + \left(\frac{M_{Edy}}{M_{Rdy}} \right)^a \leq 1,0$$

Methods of analysis

Lateral buckling of beams

No lateral buckling if:

- persistent situations: $\frac{l_{of}}{b} \leq \frac{50}{(h/b)^{1/3}}$ and $h/b \leq 2,5$
- transient situations: $\frac{l_{of}}{b} \leq \frac{70}{(h/b)^{1/3}}$ and $h/b \leq 3,5$

where:

- l_{of} is the distance between torsional restraints
- h is the total depth of beam in central part of l_{of}
- b is the width of compression flange



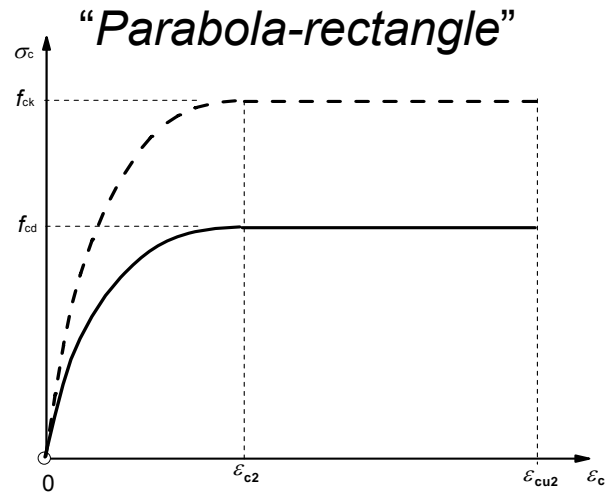
Bending with or without axial force

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Concrete design stress - strain relations (3.1.5 and 3.1.7) for section analysis



$$\sigma_c = f_{cd} \left[1 - \left(1 - \frac{\epsilon_c}{\epsilon_{c2}} \right)^n \right] \text{ for } 0 \leq \epsilon_c < \epsilon_{c2}$$

$$\sigma_c = f_{cd} \text{ for } \epsilon_{c2} \leq \epsilon_c \leq \epsilon_{cu2}$$

$$n = 1,4 + 23,4 [(90 - f_{ck})/100]^4$$

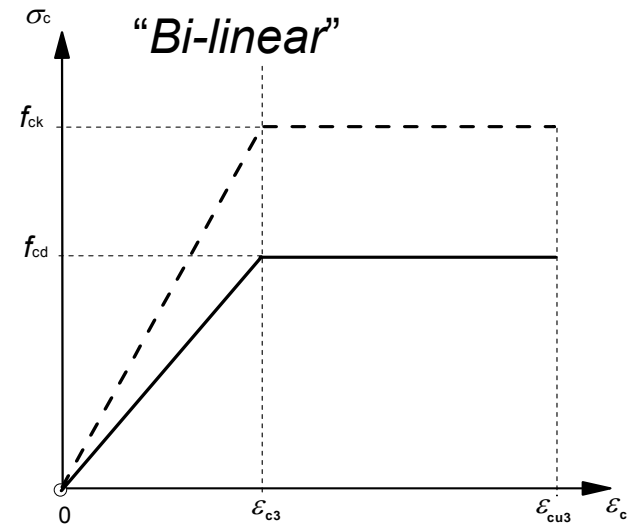
for $f_{ck} \geq 50$ MPa otherwise 2,0

$$\epsilon_{c2} (‰) = 2,0 + 0,085(f_{ck} - 50)^{0,53}$$

for $f_{ck} \geq 50$ MPa otherwise 2,0

$$\epsilon_{cu2} (‰) = 2,6 + 35 [(90 - f_{ck})/100]^4$$

for $f_{ck} \geq 50$ MPa otherwise 3,5



$$\epsilon_{c3} (‰) = 1,75 + 0,55 [(f_{ck} - 50)/40]$$

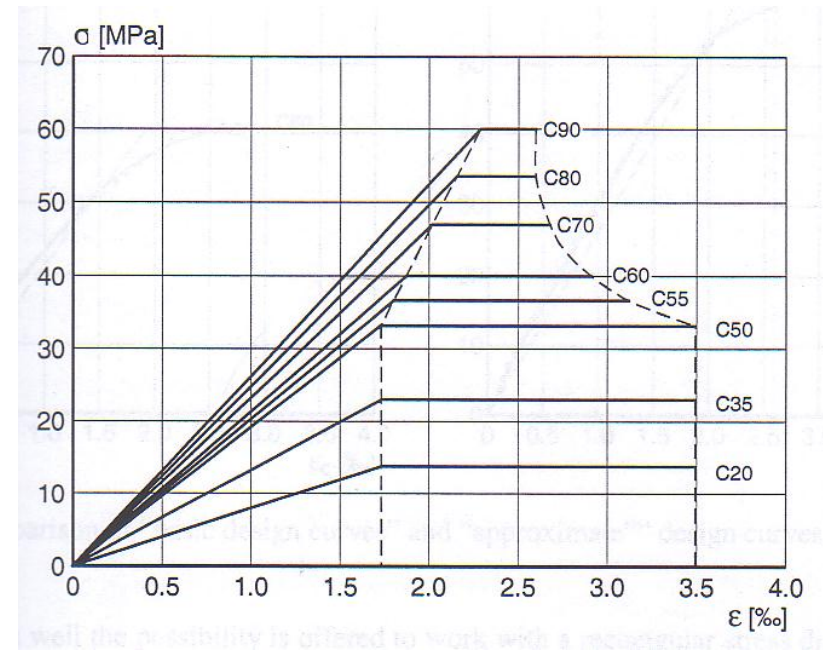
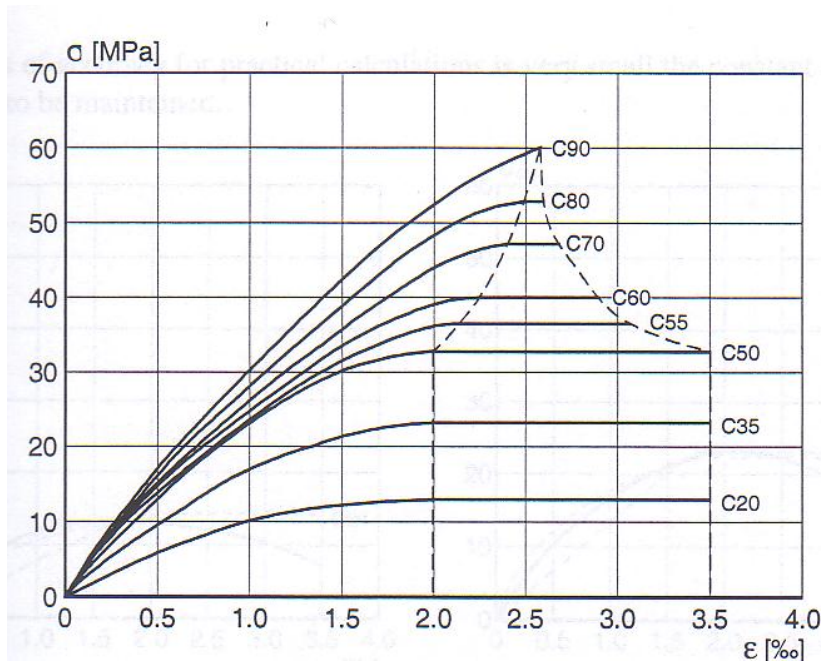
for $f_{ck} \geq 50$ MPa otherwise 1,75

$$\epsilon_{cu3} (‰) = 2,6 + 35 [(90 - f_{ck})/100]^4$$

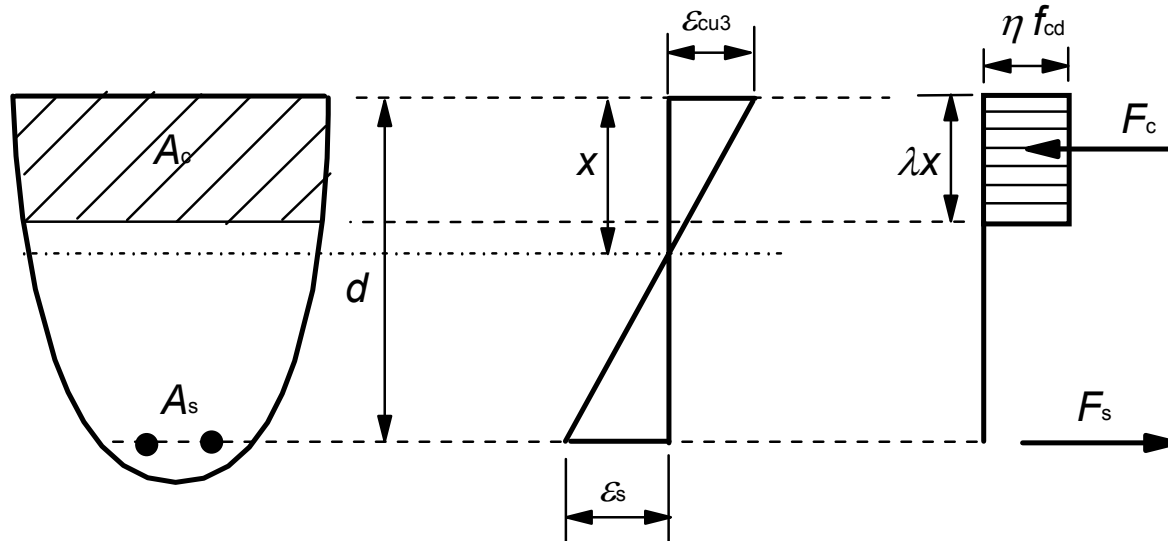
for $f_{ck} \geq 50$ MPa otherwise 3,5

Concrete design stress strain relations for different strength classes

- Higher concrete strength shows more brittle behaviour, reflected by shorter horizontal branche



Simplified concrete design stress block



$$\lambda = 0,8 \quad \text{for } f_{ck} \leq 50 \text{ MPa}$$

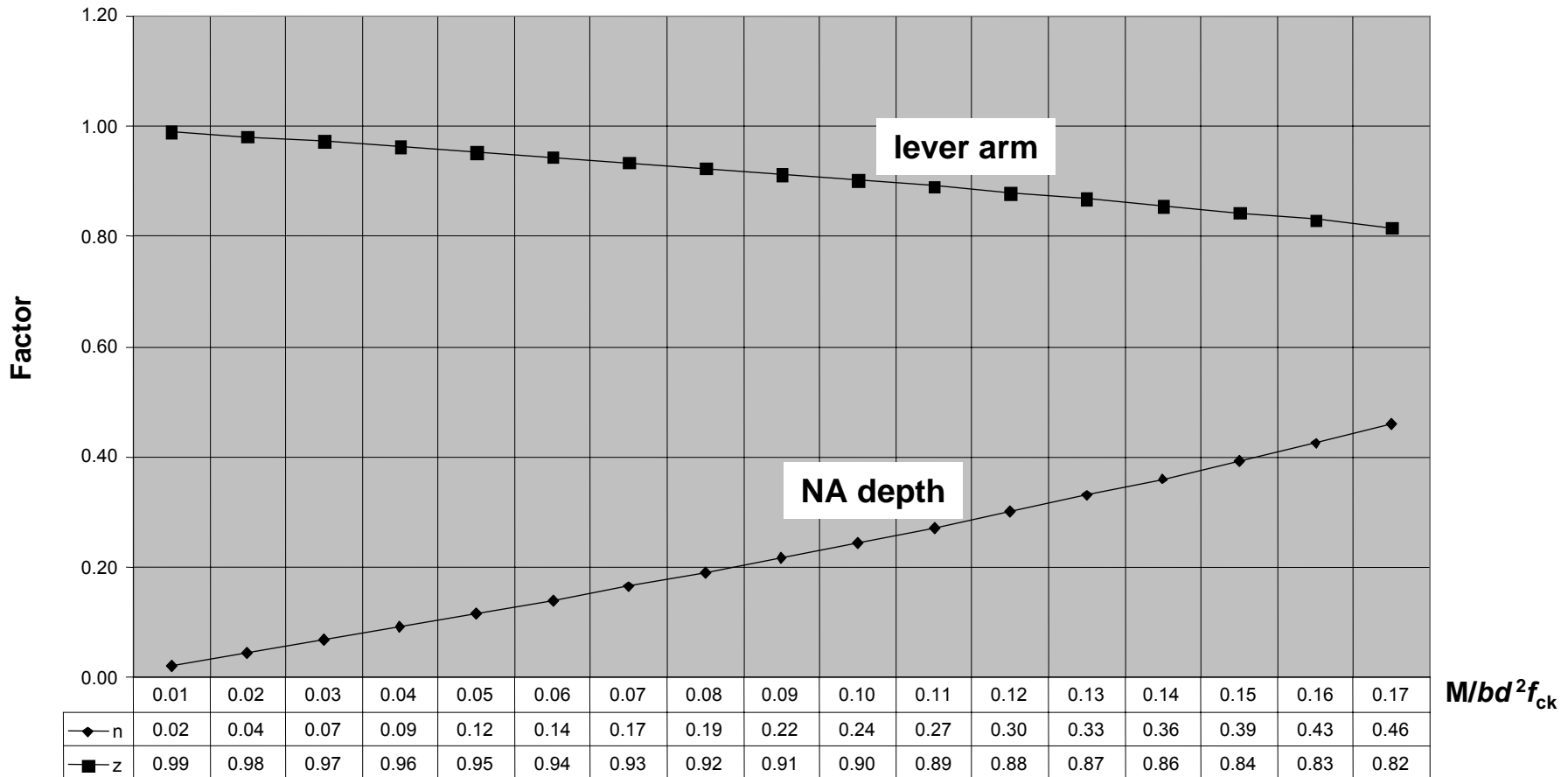
$$= 0,8 - \frac{(f_{ck} - 50)}{400} \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa}$$

$$\eta = 1,0 \quad \text{for } f_{ck} \leq 50 \text{ MPa}$$

$$= 1,0 - (f_{ck} - 50)/200 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa}$$

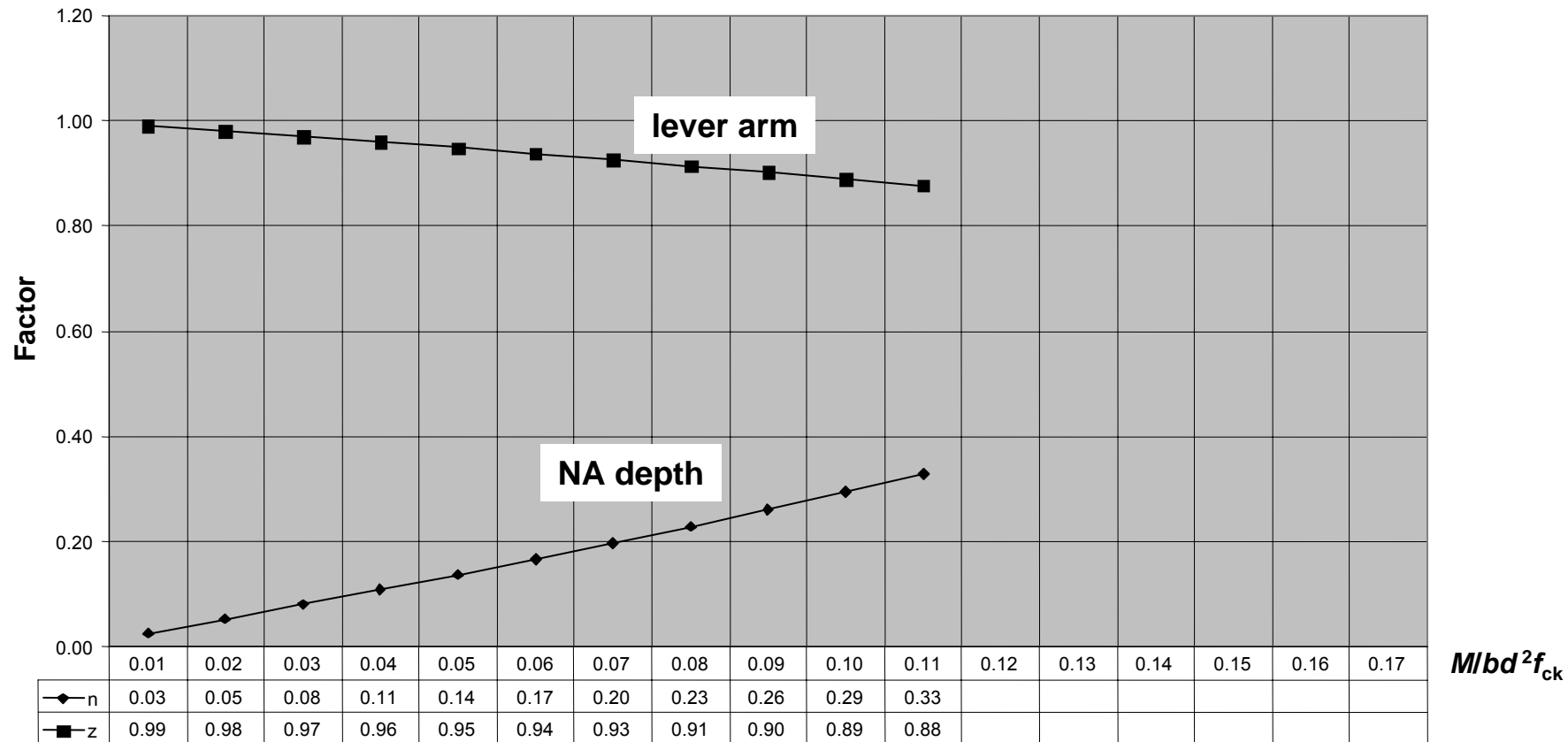
Simplified factors for flexure (1)

Factors for NA depth (n) and lever arm ($=z$) for concrete grade ≤ 50 MPa

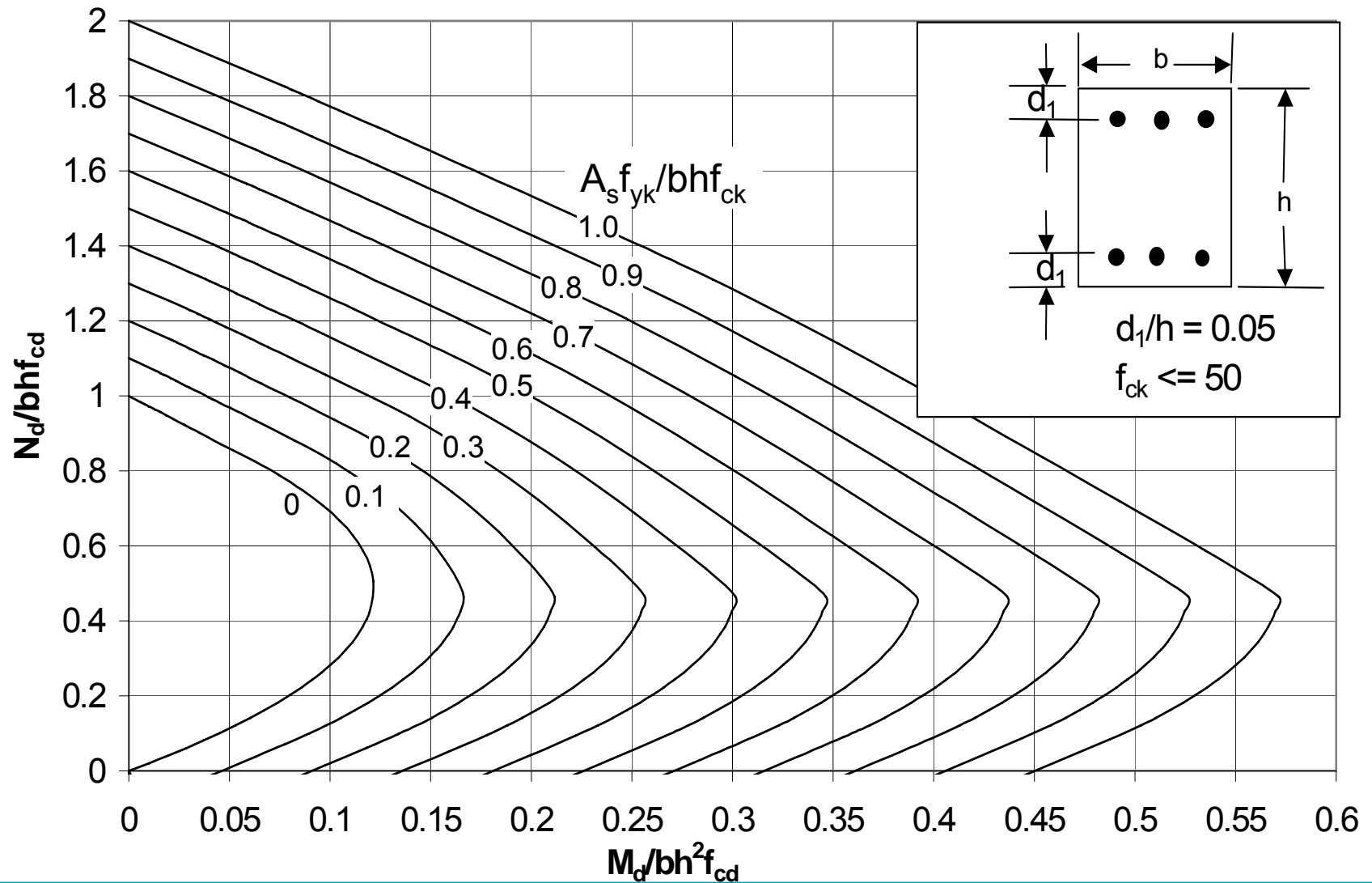


Simplified factors for flexure (2)

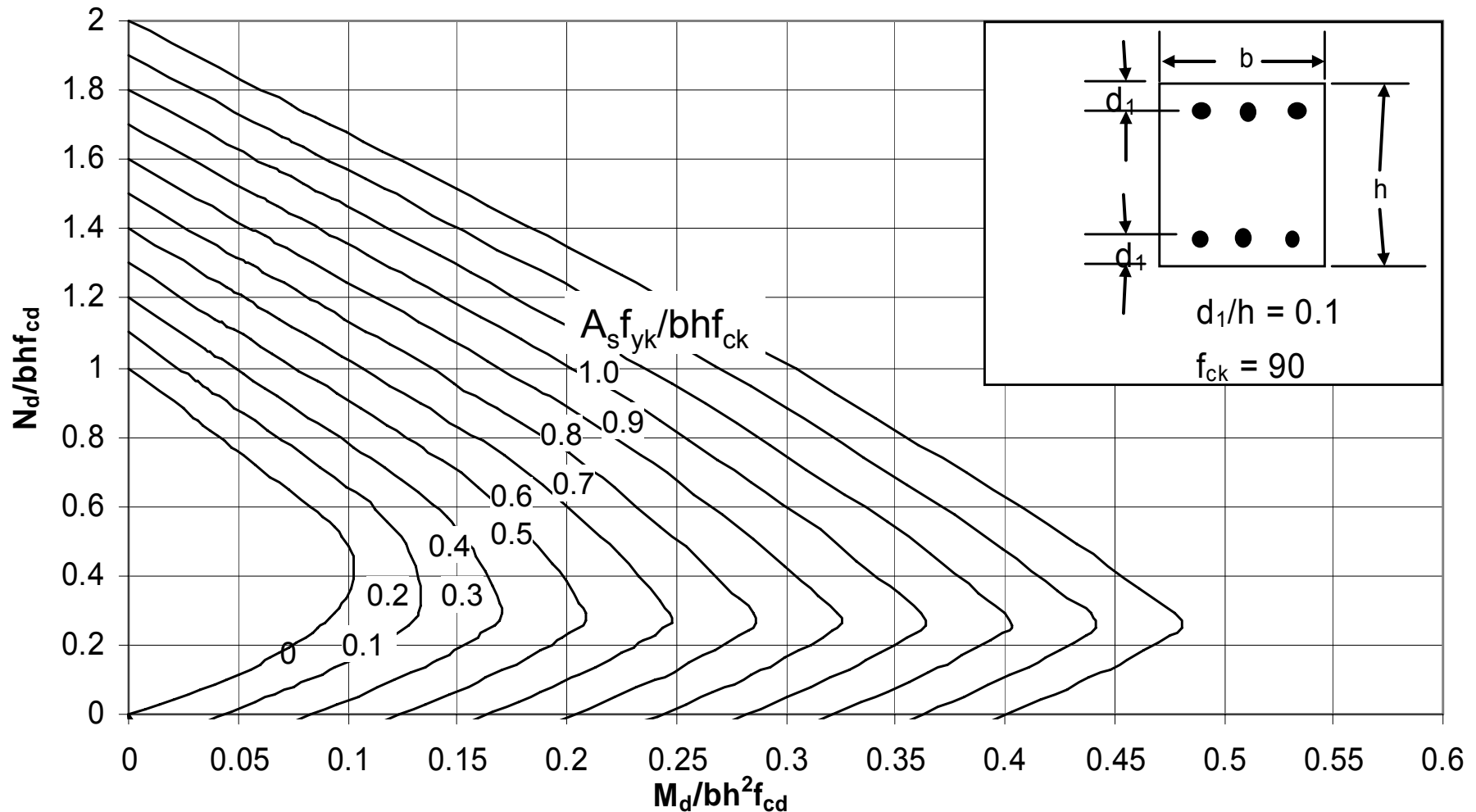
Factors for NA depth ($=n$) and lever arm ($=z$) for concrete grade 70 MPa



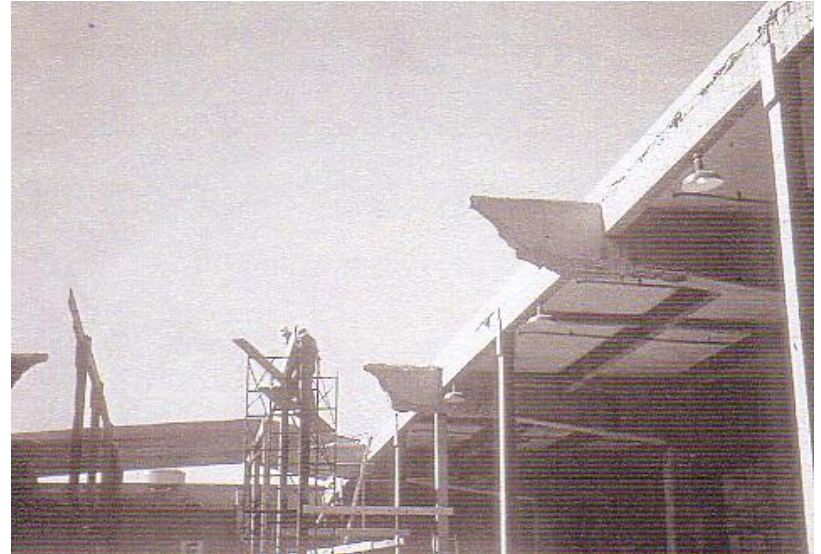
Column design chart for $f_{ck} \leq 50$ MPa



Column design chart for $f_{ck} = 70$ MPa



Shear



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22 February 2008

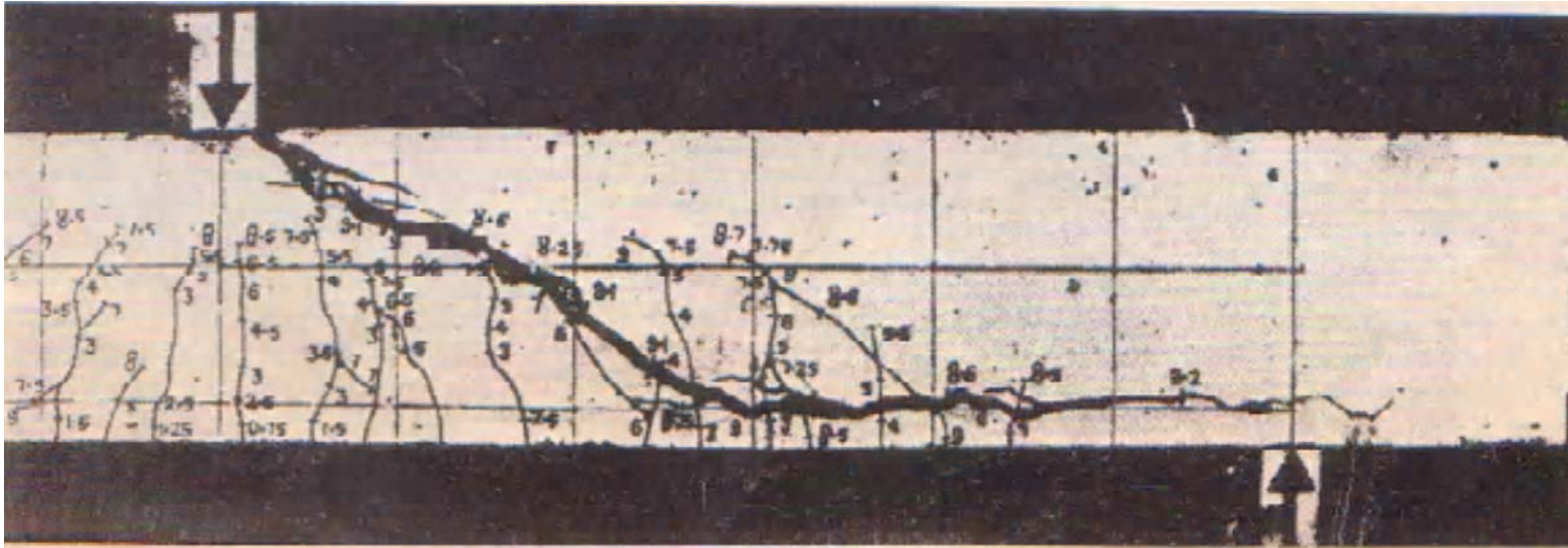
Principles of shear control in EC-2

Until a certain shear force $V_{Rd,c}$ no calculated shear reinforcement is necessary (only in beams minimum shear reinforcement is prescribed)

If the design shear force is larger than this value $V_{Rd,c}$ shear reinforcement is necessary for the full design shear force. This shear reinforcement is calculated with the variable inclination truss analogy. To this aim the strut inclination may be chosen between two values (recommended range $1 \leq \cot \theta \leq 2,5$)

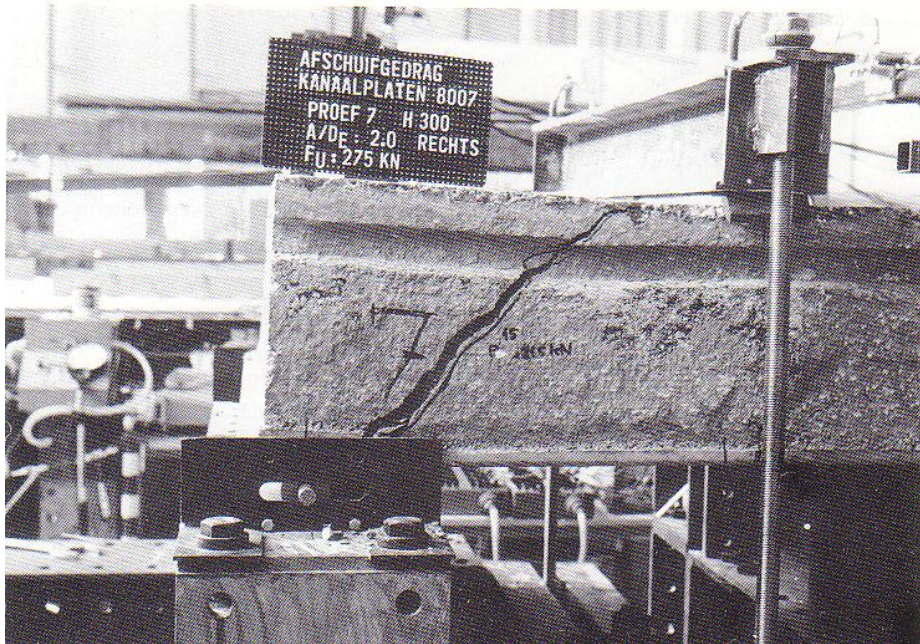
The shear reinforcement may not exceed a defined maximum value to ensure yielding of the shear reinforcement

Concrete slabs without shear reinforcement



Shear resistance $V_{Rd,c}$ governed by shear flexure failure: shear crack develops from flexural crack

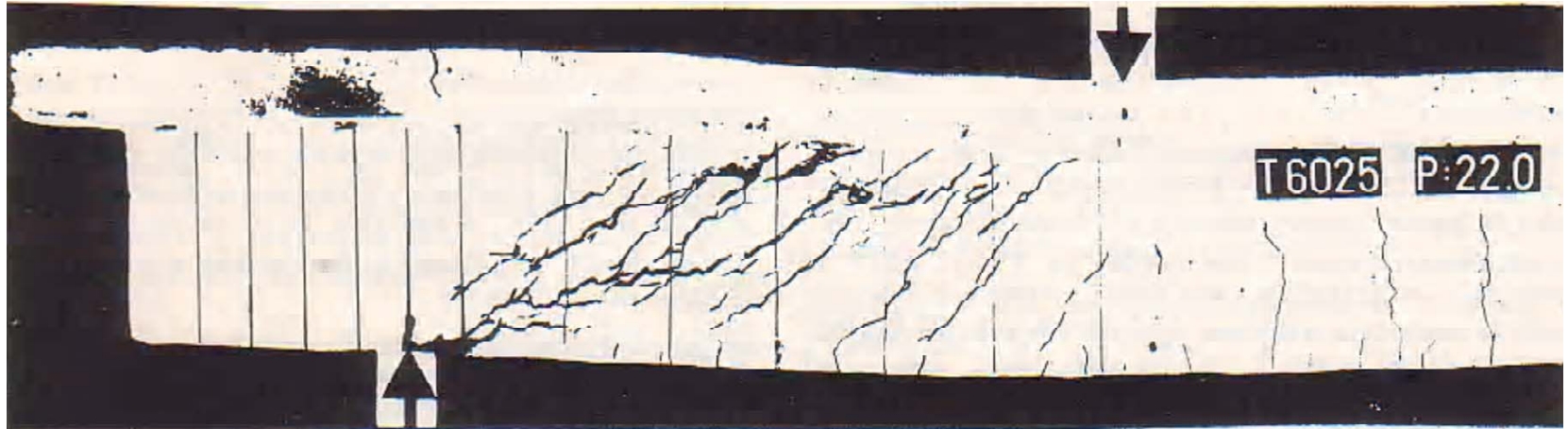
Concrete slabs without shear reinforcement



Prestressed hollow core slab

Shear resistance $V_{Rd,C}$ governed by shear tension failure:
crack occurs in web in region uncracked in flexure

Concrete beam reinforced in shear



Shear failure introduced by yielding of stirrups,
followed by strut rotation until web crushing

Principle of variable truss action

Approach “Variable inclination struts”:
a realistic

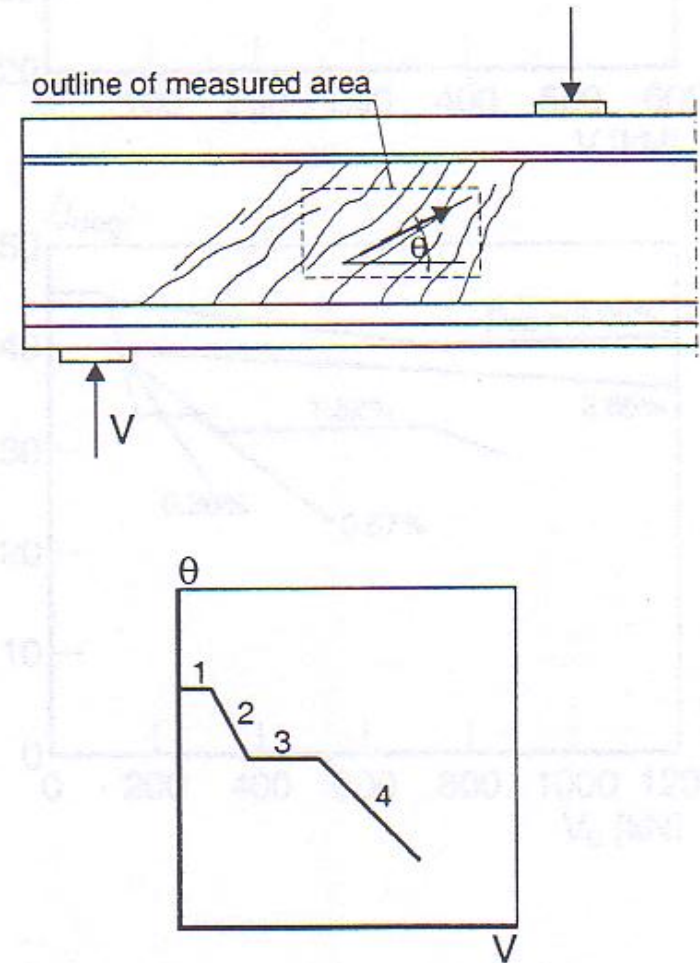
Stage 1: web uncracked in shear

Stage 2: inclined cracks occur

Stage 3: stabilized inclined cracks

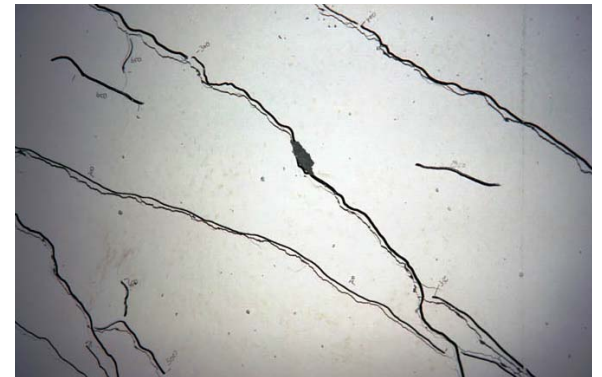
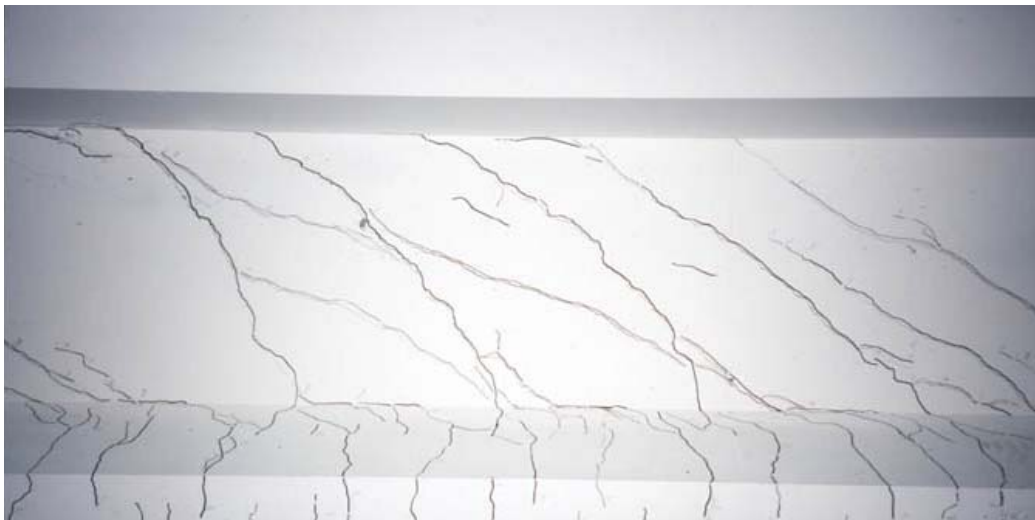
Stage 4: yielding of stirrups,
further rotation, finally
web crushing

Strut rotation as measured in tests
(TU Delft)



Principles of variable angle truss

Strut rotation, followed by new cracks under lower angle, even in high strength concrete (Tests TU Delft)



Web crushing in concrete beam



Web crushing provides maximum to shear resistance

At web crushing:

$$V_{Rd,max} = b_w z v f_{cd} / (\cot\theta + \tan\theta)$$

Advantage of variable angle truss analogy

-Freedom of design:

- low angle θ leads to low shear reinforcement
- High angle θ leads to thin webs, saving concrete and dead weight

Optimum choice depends on type of structure

- Transparent equilibrium model, easy in use

Shear design value under which no shear reinforcement is necessary in elements unreinforced in shear (general limit)

$$V_{Rd,c} = C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} b_w d$$

$C_{Rd,c}$	<i>coefficient derived from tests (recommended 0,12)</i>
k	<i>size factor = $1 + \sqrt{(200/d)}$ with d in meter</i>
ρ_l	<i>longitudinal reinforcement ratio ($\leq 0,02$)</i>
f_{ck}	<i>characteristic concrete compressive strength</i>
b_w	<i>smallest web width</i>
d	<i>effective height of cross section</i>

Shear design value under which no shear reinforcement is necessary in elements unreinforced in shear (general limit)

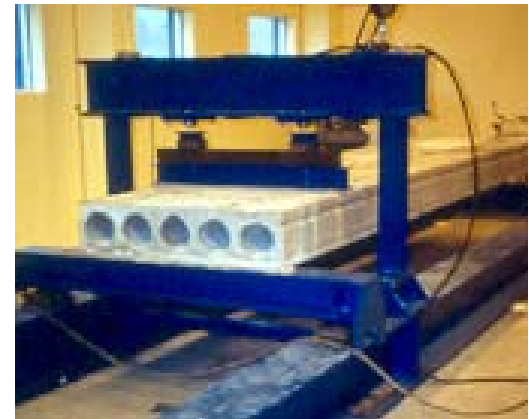
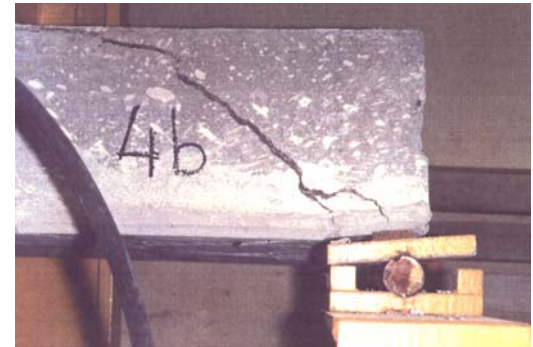
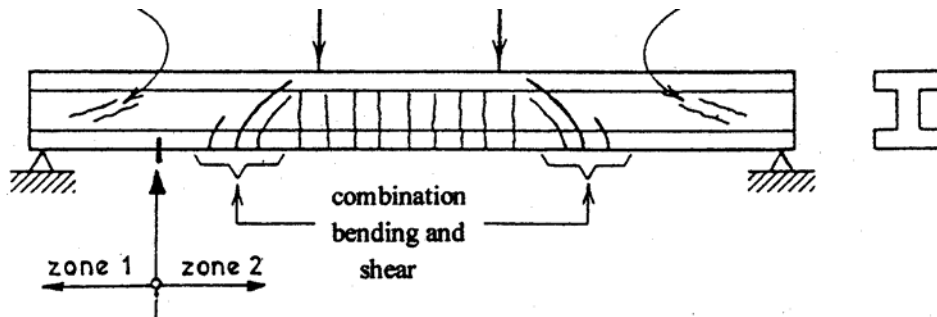
Minimum value for $V_{Rd,c}$:

$$V_{Rd,c} = v_{min} b_w d$$

Values for v_{min} (N/mm²)

	d=200	d=400	d=600	d=800
C20	0,44	0,35	0,25	0,29
C40	0,63	0,49	0,44	0,41
C60	0,77	0,61	0,54	0,50
C80	0,89	0,70	0,62	0,58

Shear design value under which no shear reinforcement is necessary in elements unreinforced in shear (special case of shear tension)



Special case of shear tension (example hollow core slabs)

$$V_{Rd,c} = \frac{I \cdot b_w}{S} \sqrt{(f_{ctd})^2 + \alpha_l \sigma_{cp} f_{ctd}}$$

I *moment of inertia*

b_w *smallest web width*

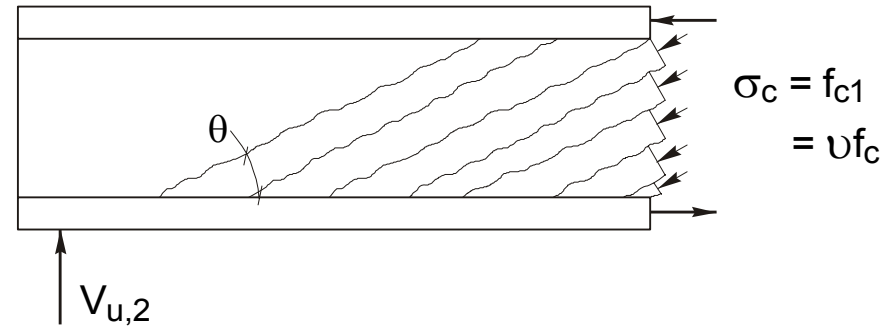
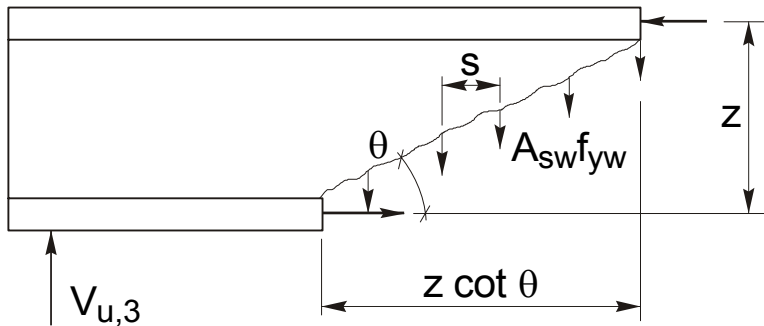
S *section modulus*

f_{ctd} *design tensile strength of concrete*

α_l *reduction factor for prestress in case of prestressing strands or wires in ends of member*

σ_{cp} *concrete compressive stress at centroidal axis ifor for fully developed prestress*

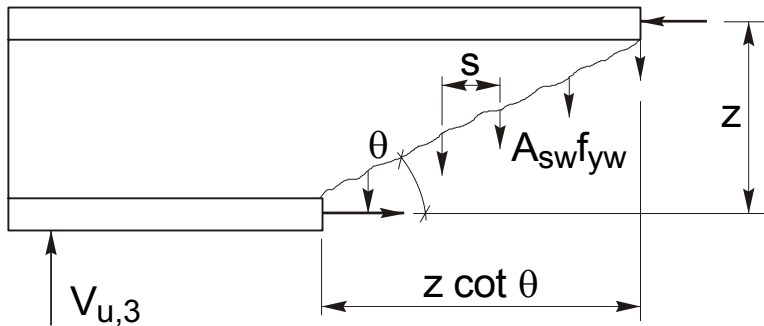
Design of members if shear reinforcement is needed ($V_{E,d} > V_{Rd,c}$)



For most cases:

- Assume $\cot \theta = 2,5$ ($\theta = 21,8^\circ$)
- Calculate necessary shear reinforcement
- Check if web crushing capacity is not exceeded ($V_{Ed} > V_{Rd,s}$)
- If web crushing capacity is exceeded, enlarge web width or calculate the value of $\cot \theta$ for which $V_{Ed} = V_{Rd,c}$ and repeat the calculation

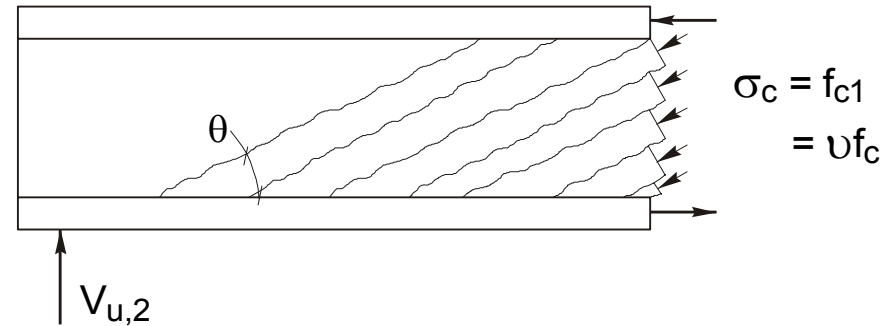
Upper limit of shear capacity reached due to web crushing



For yielding shear reinforcement:

$$V_{Rd,s} = (A_{sw}/s) z f_{ywd} \cot \theta$$

θ from 45° to $21,8^\circ$
2,5 times larger capacity

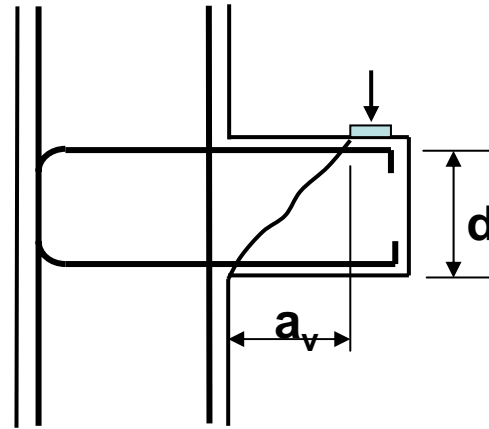
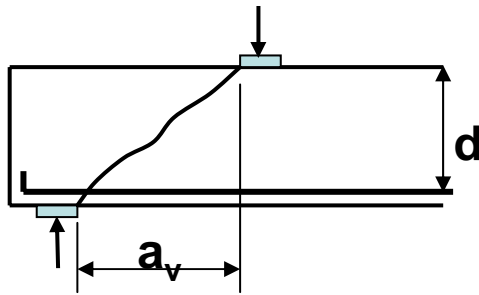


At web crushing:

$$V_{Rd,max} = b_w z v f_{cd} / (\cot \theta + \tan \theta)$$

θ from $21,8^\circ$ to 45°
1,45 times larger capacity

Special case of loads near to supports



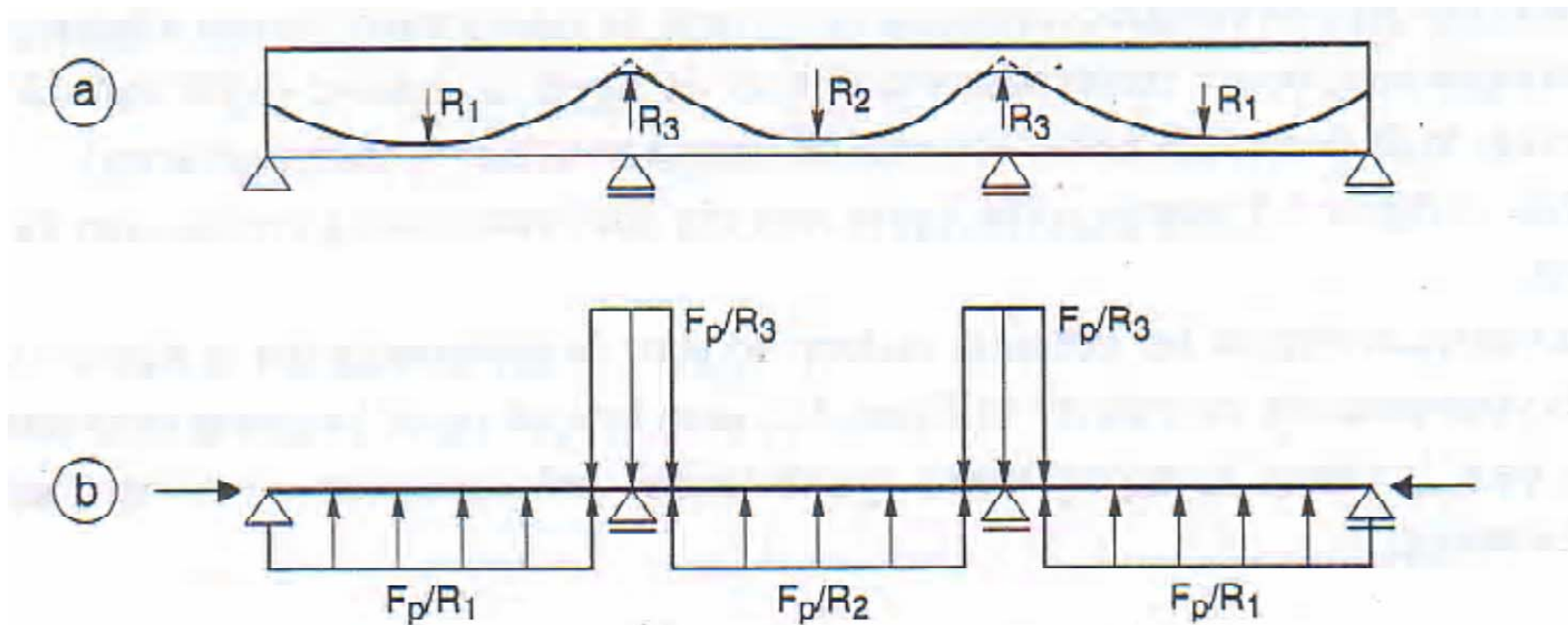
For $a_v \leq 2d$ the contribution of the point load to the shear force V_{Ed} may be reduced by a factor $a_v/2d$ where $0.5 \leq a_v \leq 2d$ provided that the longitudinal reinforcement is fully anchored at the support. However, the condition

$$V_{Ed} \leq 0,5b_w d v f_{cd}$$

should always be fulfilled

Influence of prestressing on shear resistance (1)

1. Prestressing introduces a set of loads on the beam



Influence of prestressing on shear resistance (2)

Prestressing increases the load $V_{Rd,c}$ below which no calculated shear reinforcement is required

$$V_{Rd,c} = [C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}] b_w d$$

k_1 *coefficient, with recommended value 0,15*
 σ_{cp} *concrete compressive stress at centroidal axis due to axial loading or prestressing*

Influence of prestressing on shear resistance (3)

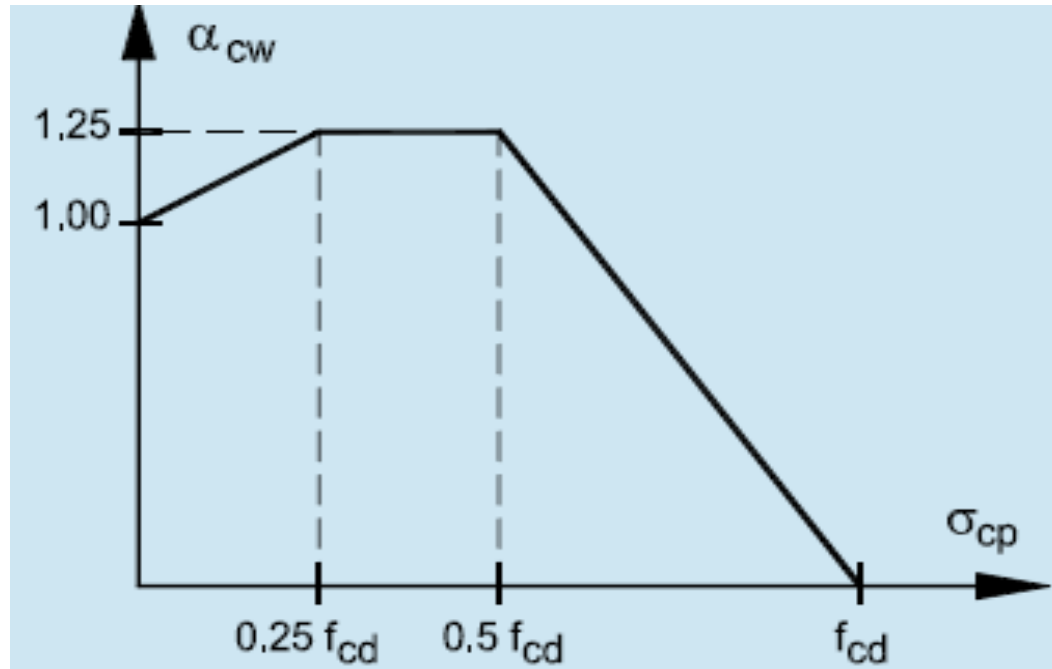
1. Prestressing increases the web crushing capacity

$$V_{Rd,max} = \alpha_{cw} b_w z v f_{cd} / (\cot \theta + \tan \theta)$$

α_{cw} factor depending on prestressing force

$\alpha_{cw} =$	1	for non prestressed structures
	$(1 + \sigma_{cp} / f_{cd})$	for $0,25 < \sigma_{cp} < 0,25 f_{cd}$
	1,25	for $0,25 f_{cd} < \sigma_{cp} < 0,5 f_{cd}$
	$2,5(1 - \sigma_{cp} / f_{cd})$	for $0,5 f_{cd} < \sigma_{cp} < 1,0 f_{cd}$

Increase of web crushing capacity by prestressing (4)

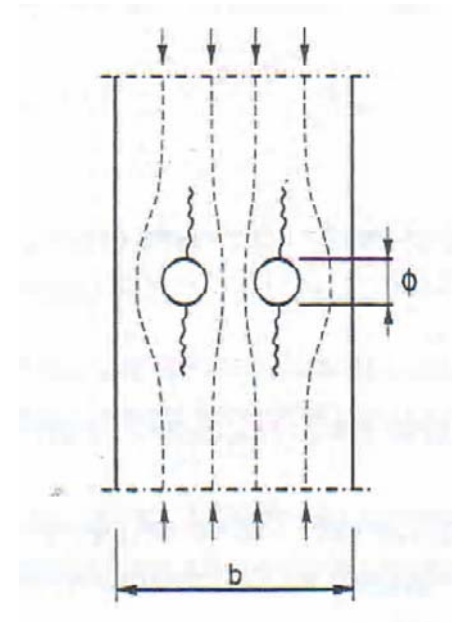
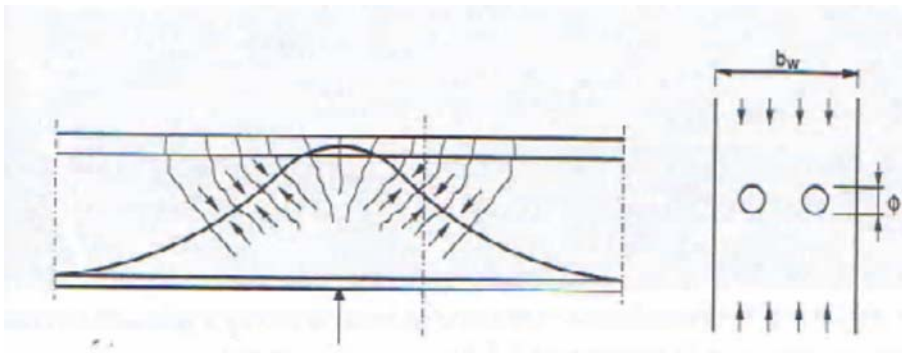


Influence of prestressing on shear resistance (4)

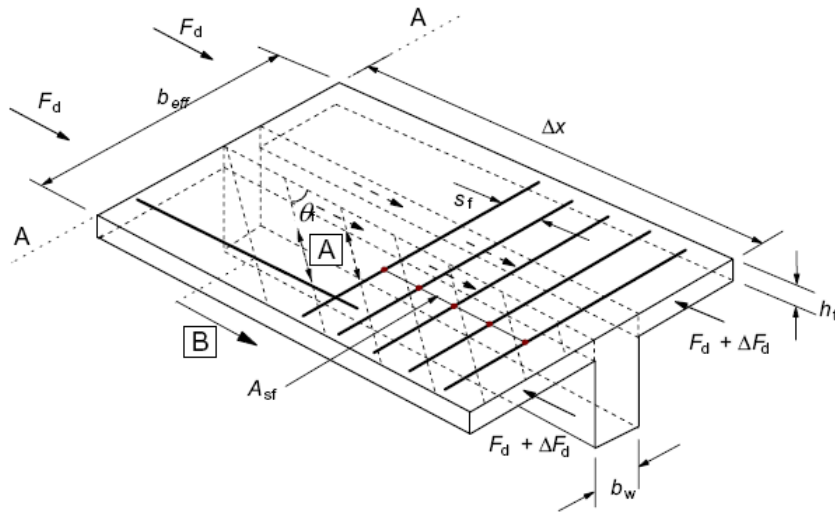
Reducing effect of prestressing duct (with or without tendon) on web crushing capacity

Grouted ducts $b_{w,nom} = b_w - \Sigma\phi$

Ungrouted ducts $b_{w,nom} = b_w - 1,2 \Sigma\phi$



Shear between web and flanges of T-sections



Strut angle θ :

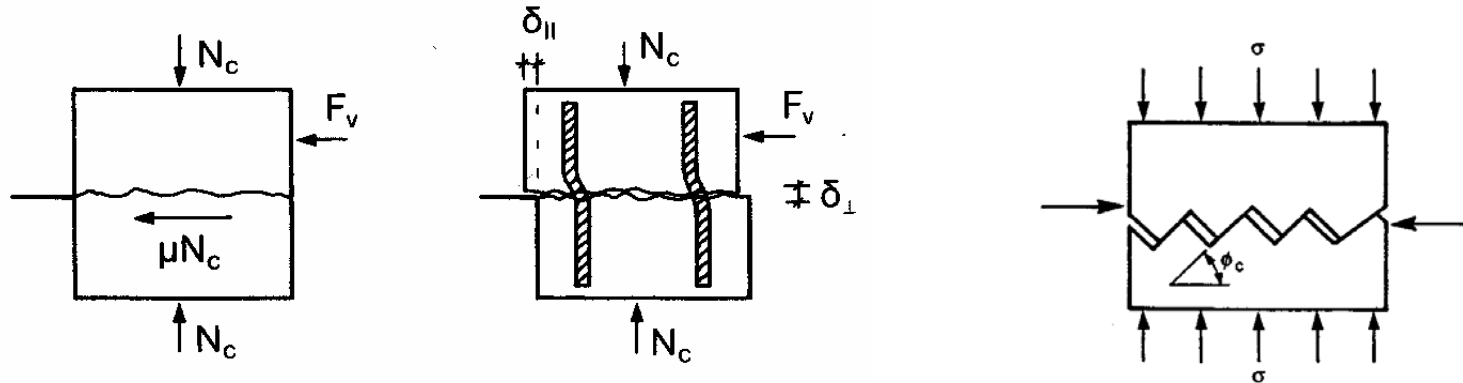
$1,0 \leq \cot \theta_f \leq 2,0$ for compression flanges ($45^\circ \geq \theta_f \geq 26,5^\circ$)

$1,0 \leq \cot \theta_f \leq 1,25$ for tension flanges ($45^\circ \geq \theta_f \geq 38,6^\circ$)

No transverse tension ties required if shear stress in interface

$$v_{Ed} = \Delta F_d / (h_f \cdot \Delta x) \leq k f_{ctd} \quad (\text{recommended } k = 0,4)$$

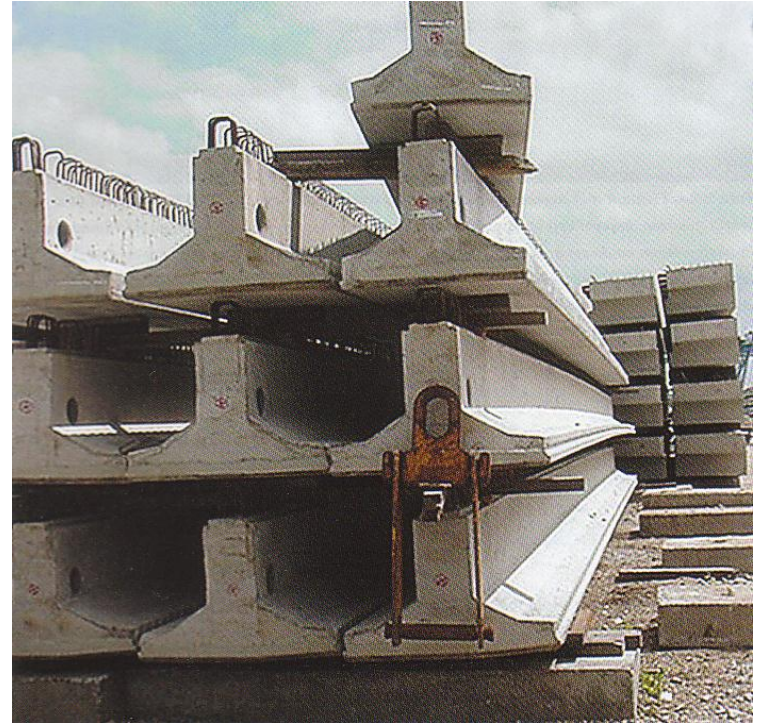
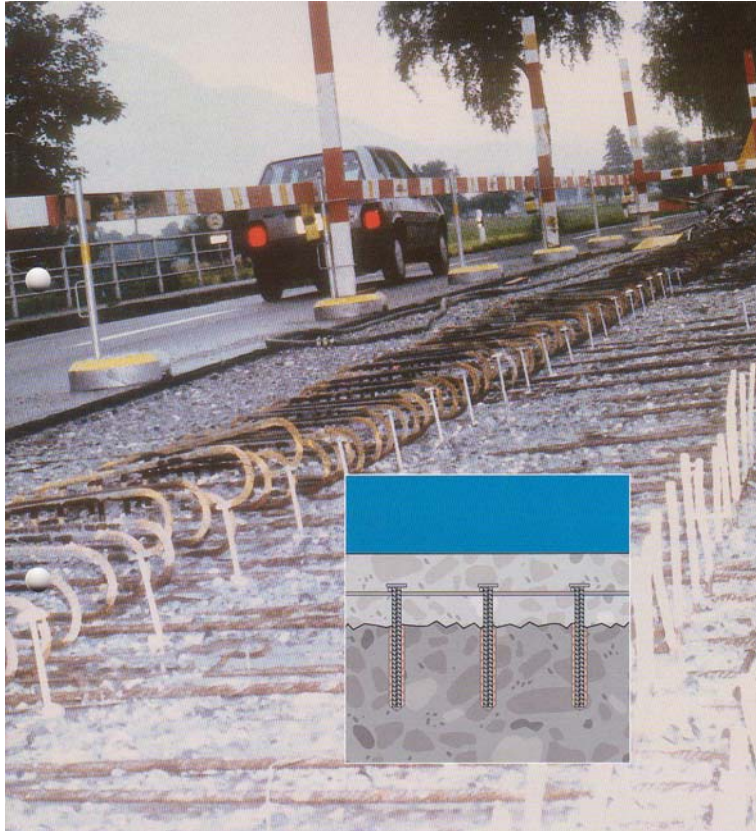
Shear at the interface between concretes cast at different times



Interface shear models based on shear friction principle

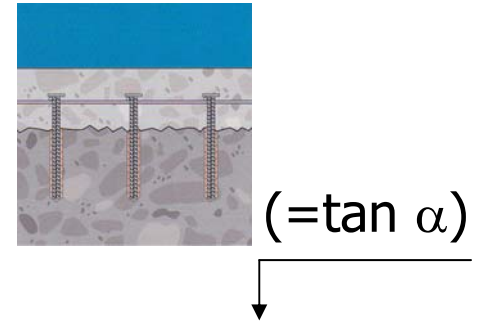
$$V_{Rdi} = c f_{ctd} + \mu \sigma_n + \rho f_{yd} (\mu \sin \alpha + \cos \alpha) \leq 0,5 v f_{cd}$$

Shear at the interface between concretes cast at different times



Shear at the interface between concrete's cast at different times (Eurocode 2, Clause 6.5.2)

$$V_{Rdi} = c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} (\mu \cdot \sin \beta + \cos \beta) \leq 0,5 \cdot v \cdot f_{cd}$$



f_{ctd} = concrete design tensile strength

σ_n = eventual confining stress, not from reinforcement

ρ = reinforcement ratio

β = inclination between reinforcement and concrete surface

f_{cd} = concrete design compressive strength

$v = 0,6$ for $f_{ck} \leq 60$ MPa

$= 0,9 - f_{ck}/200 \geq 0,5$ for $f_{ck} \geq 60$ MPa

	c	μ
Very smooth	0,25	0,5
smooth	0,35	0,6
rough	0,45	0,7
indented	0,50	0,8

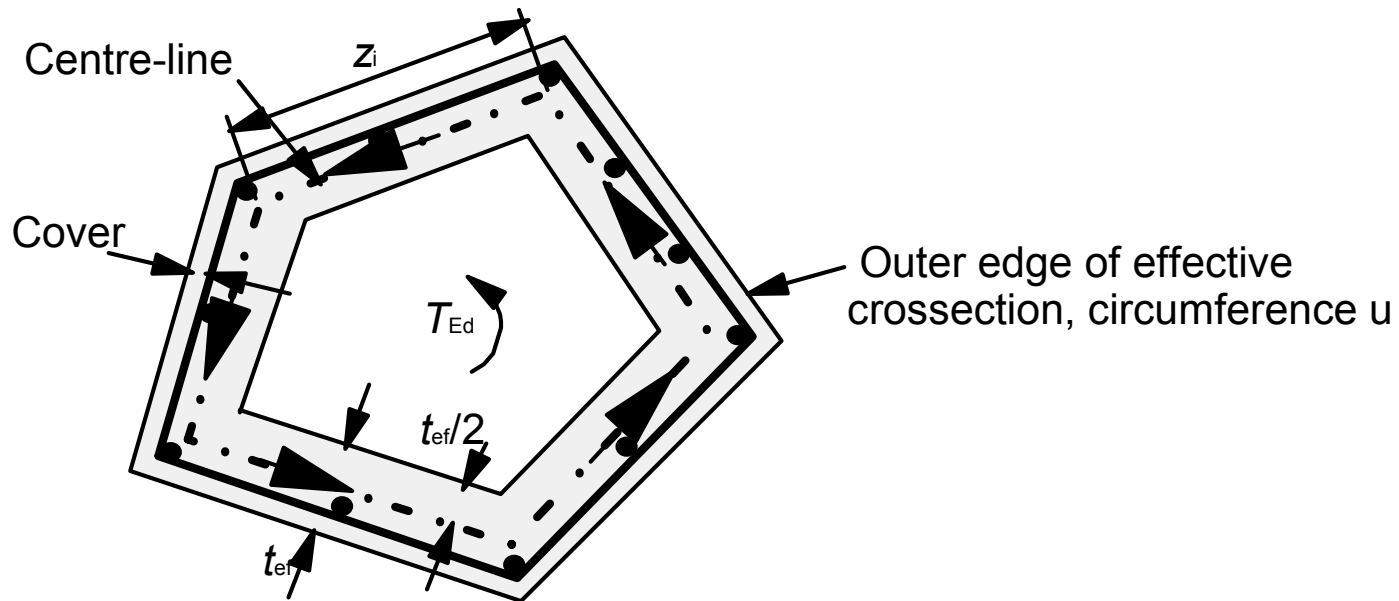
Torsion

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Modeling solid cross sections by equivalent thin-walled cross sections



Effective wall-thickness follows from $t_{ef,i} = A/u$, where;
 A = total area of cross section within outer circumference, including hollow areas
 U = outer circumference of the cross section

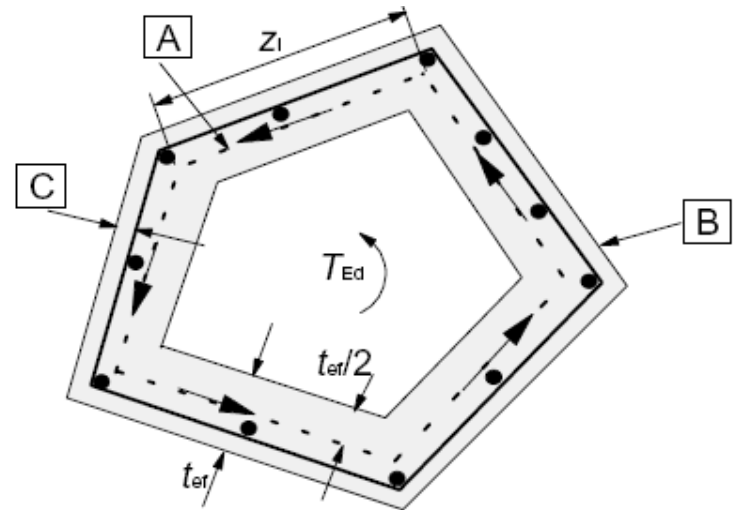
Design procedure for torsion (1)

Shear flow in any wall follows from:

$$\tau_{t,i} t_{ef,i} = \frac{T_{Ed}}{2A_k}$$

where

$\tau_{t,I}$ *torsional shear stress in wall I*
 $t_{ef,I}$ *effective wall thickness (A/u)*
 T_{Ed} *applied torsional moment*
 A_k *area enclosed by centre lines of connecting walls, including hollow areas*



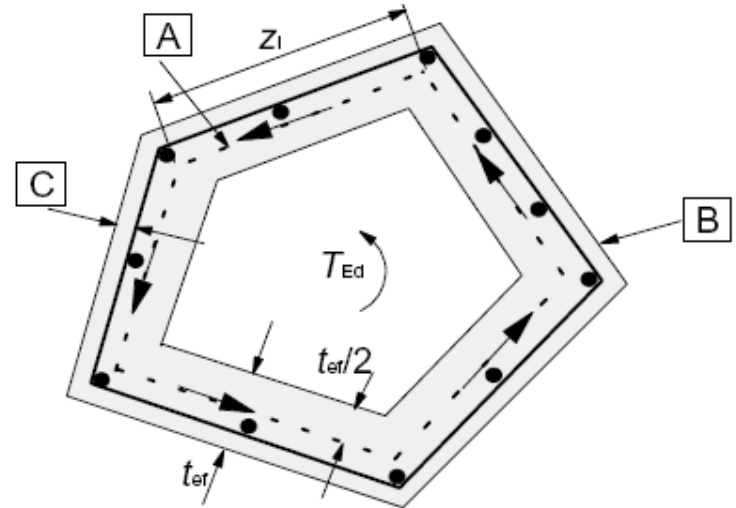
Design procedure for torsion (2)

Shear force V_{Ed} in wall i due to torsion is:

$$V_{Ed,i} = \tau_{t,i} t_{ef,i} z_i$$

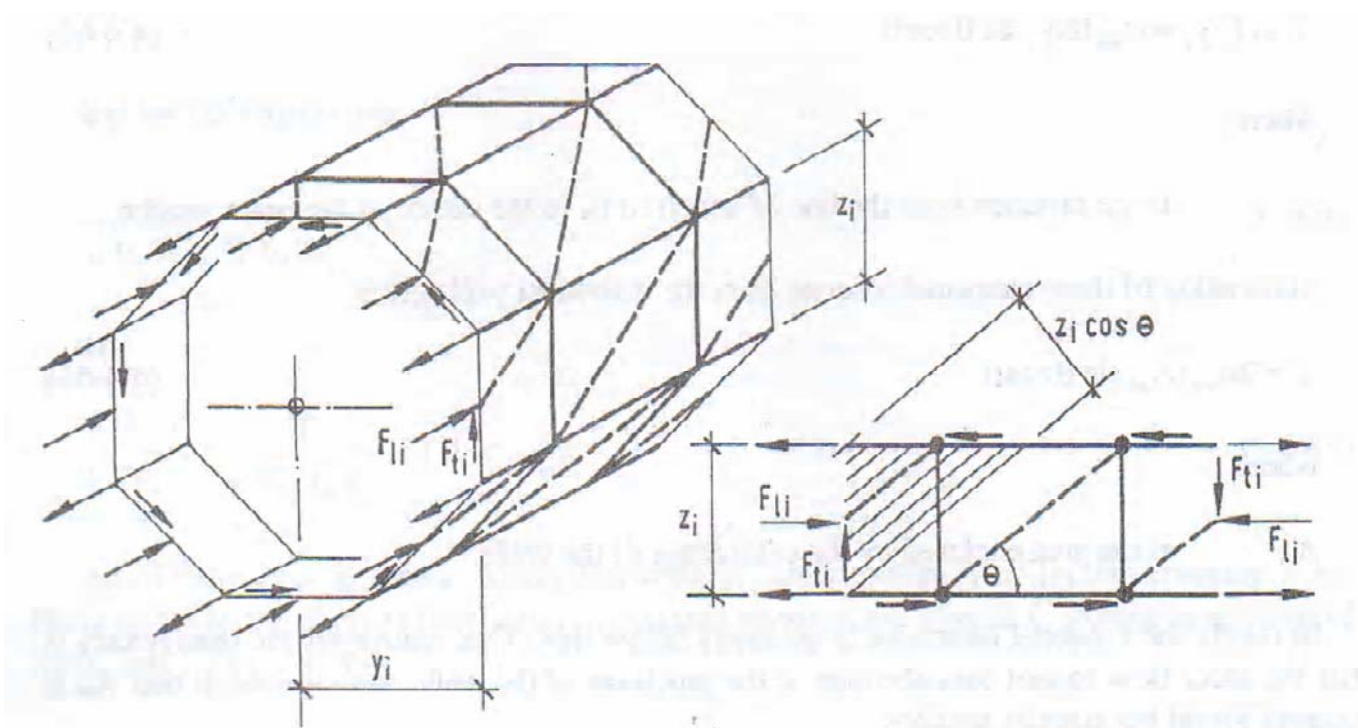
where

$\tau_{t,i}$ *torsional shear stress in wall i*
 $t_{ef,i}$ *effective wall thickness (A/u)*
 z_i *inside length of wall i defined by distance of intersection points with adjacent walls*



Design procedure for torsion (3)

The shear reinforcement in any wall can now be designed like a beam using the variable angle truss analogy, with $1 \leq \cot \theta \leq 2,5$



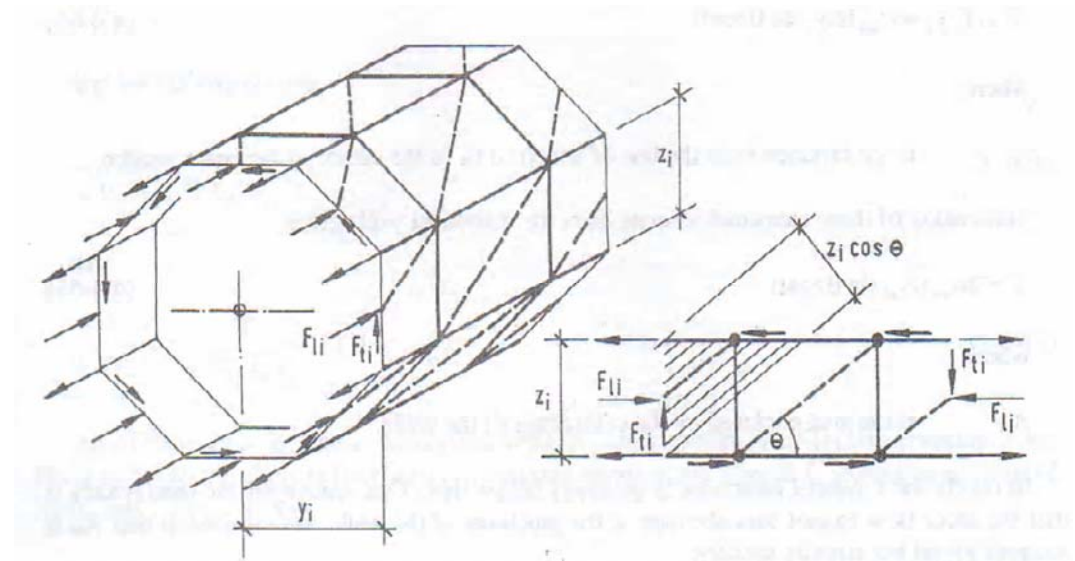
Design procedure for torsion (4)

The longitudinal reinforcement in any wall follows from:

$$\frac{\Sigma A_{sl} f_{yd}}{u_k} = \frac{T_{Ed}}{2A_k} \cot \theta$$

where

u_k perimeter of area A_k
 f_{yk} design yield stress of steel
 θ angle of compression struts



Punching shear

Prof.dr.ir. J.C. Walraven

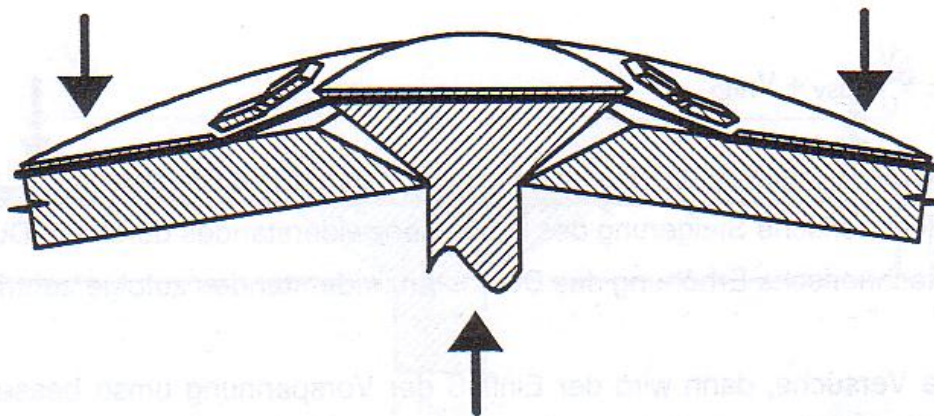
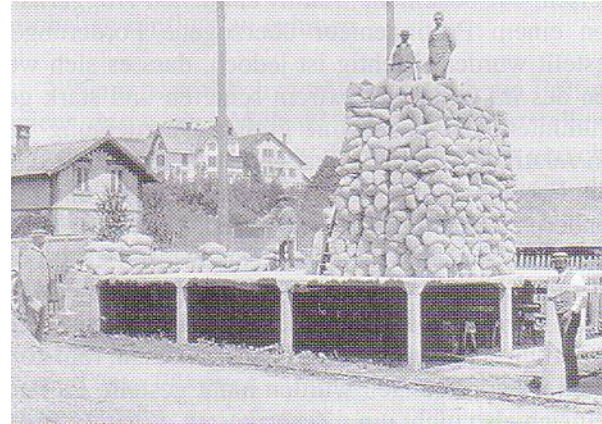
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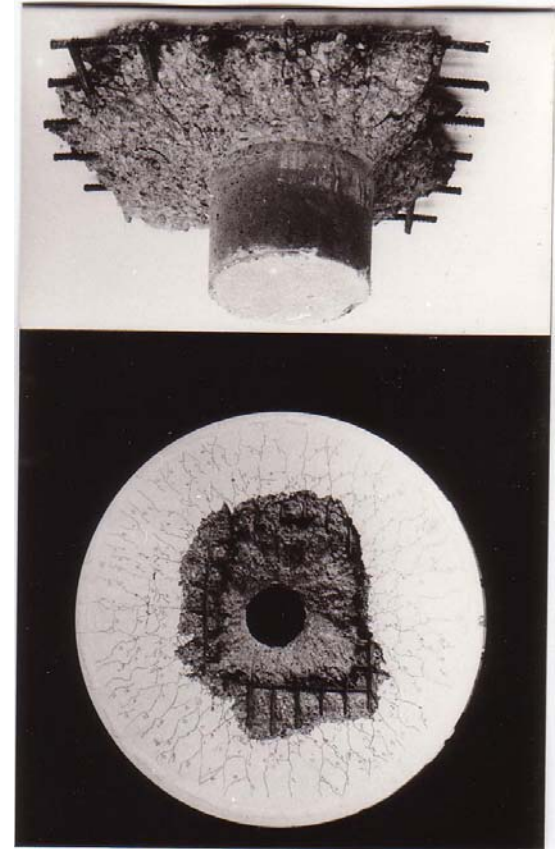
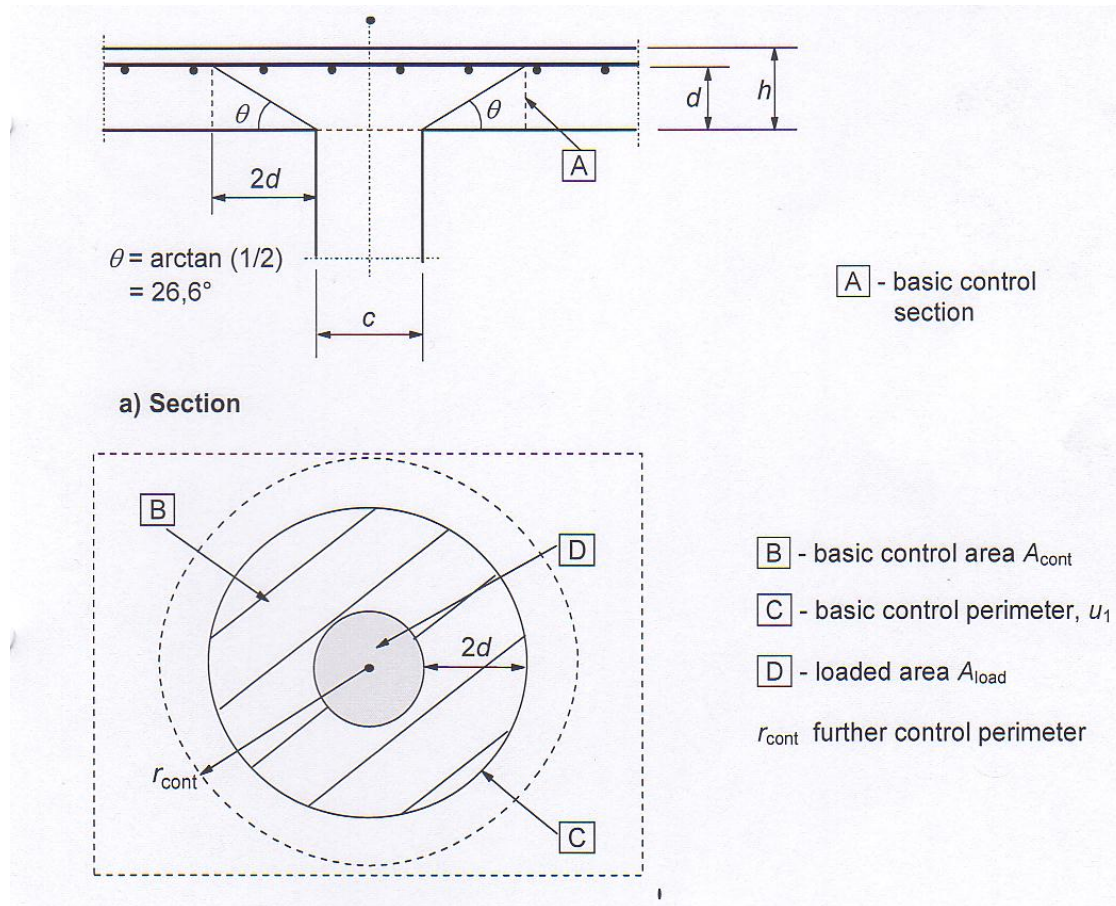
Design for punching shear

Most important aspects:

- Control perimeter
- Edge and corner columns
- Simplified versus advanced control methods

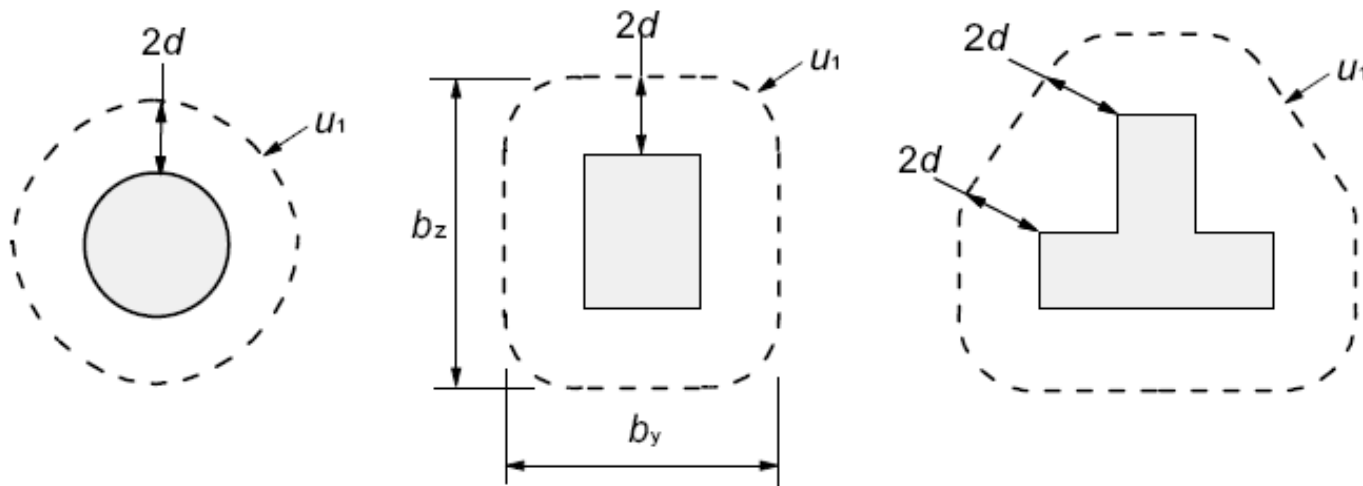


Definition of control perimeter



Definition of control perimeters

The basic control perimeter u_1 is taken at a distance $2,0d$ from the loaded area and should be constructed as to minimise its length



Limit values for design punching shear stress in design

The following limit values for the punching shear stress are used in design:

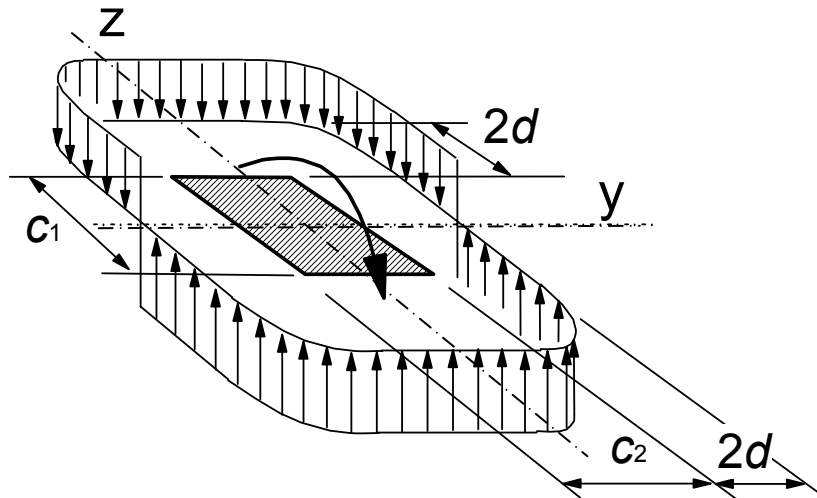
If $v_{Ed} \leq v_{Rd,c}$ no punching shear reinforcement required

where:

$$v_{Rd,c} = C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + 0,10 \sigma_{cp} \geq (v_{\min} + 0,10 \sigma_{cp})$$

How to take account of eccentricity

More sophisticated method for internal columns:



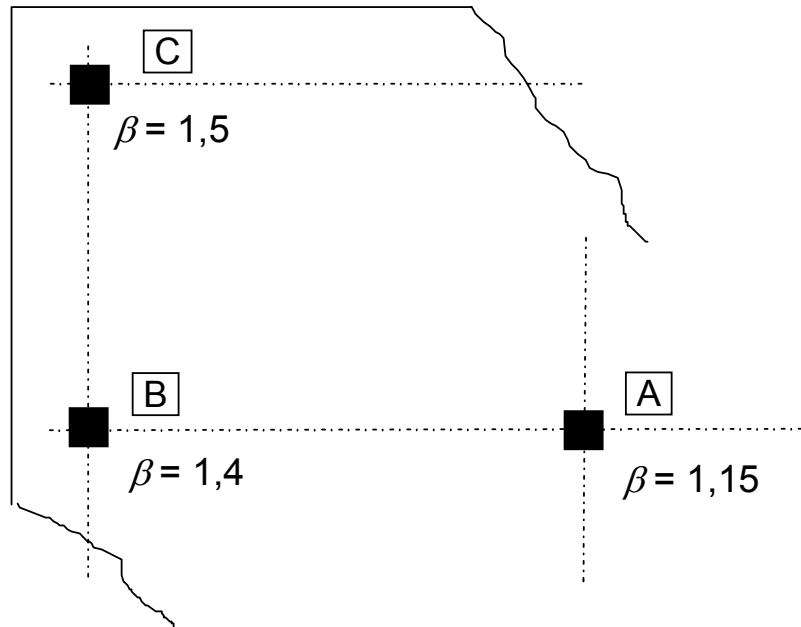
$$\beta = 1 + 1,8 \sqrt{\left(\frac{e_y}{b_z}\right)^2 + \left(\frac{e_z}{b_y}\right)^2}$$

e_y and e_z
 b_y and b_z

*eccentricities M_{Ed}/V_{Ed} along y and z axes
dimensions of control perimeter*

How to take account of eccentricity

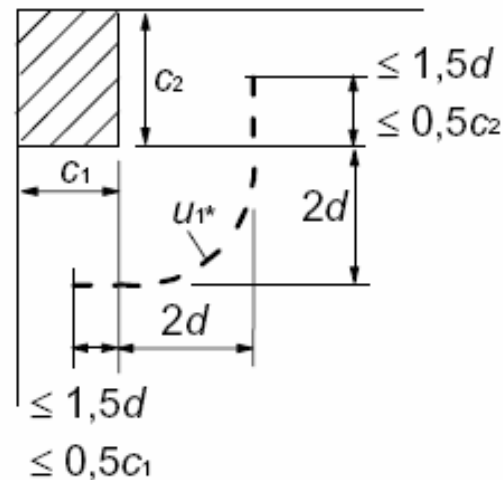
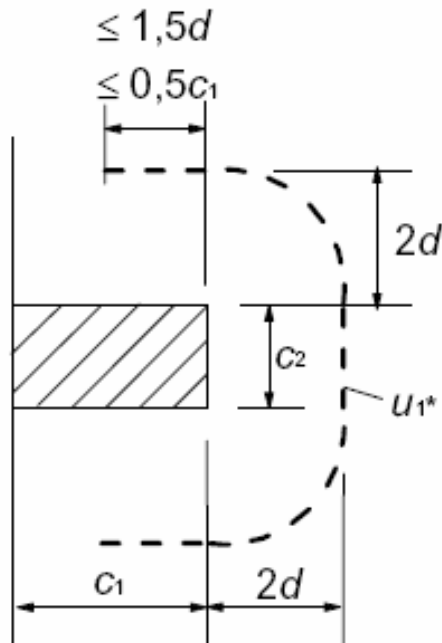
Or, how to determine β in equation $v_{Ed} = \beta \frac{V_{Ed}}{u_i d}$



For structures where lateral stability does not depend on frame action and where adjacent spans do not differ by more than 25% the approximate values for β shown below may be used:

How to take account of eccentricity

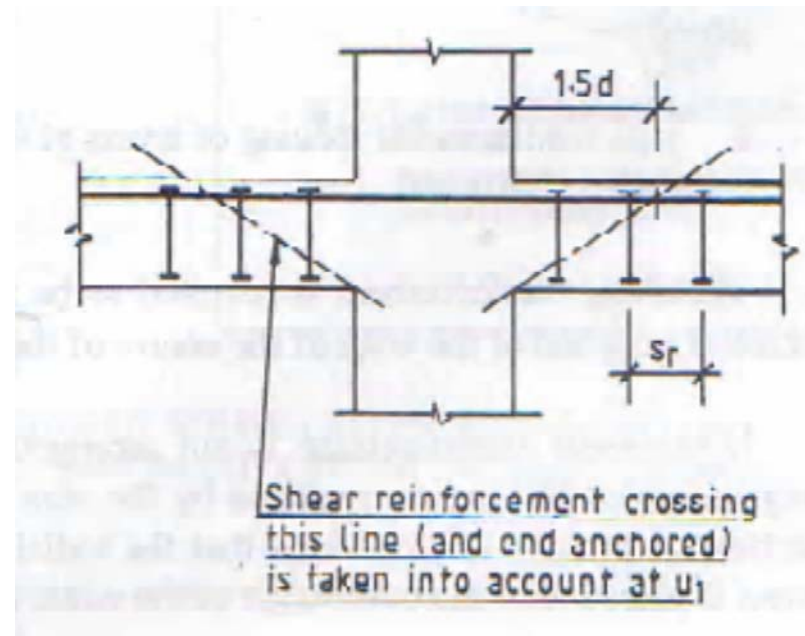
Alternative for edge and corner columns: use perimeter u_{1*} in stead of full perimeter and assume uniform distribution of punching force



Design of punching shear reinforcement

If $V_{Ed} \geq V_{Rd,c}$ shear reinforcement is required.

The steel contribution comes from the shear reinforcement crossing a surface at $1,5d$ from the edge of the loaded area, to ensure some anchorage at the upper end. The concrete component of resistance is taken 75% of the design strength of a slab without shear reinforcement



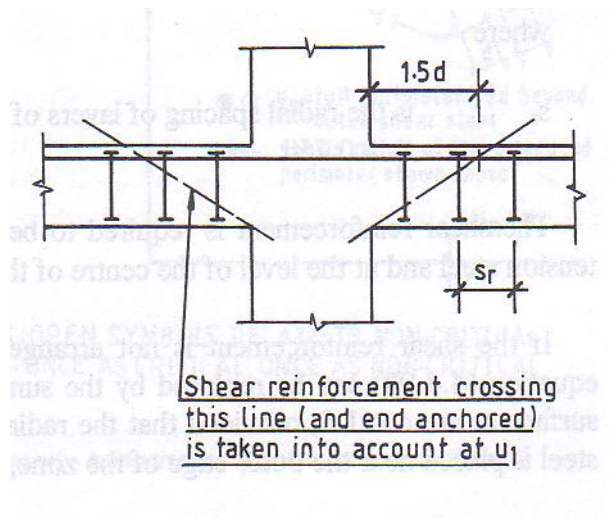
Punching shear reinforcement

Capacity with punching shear reinforcement

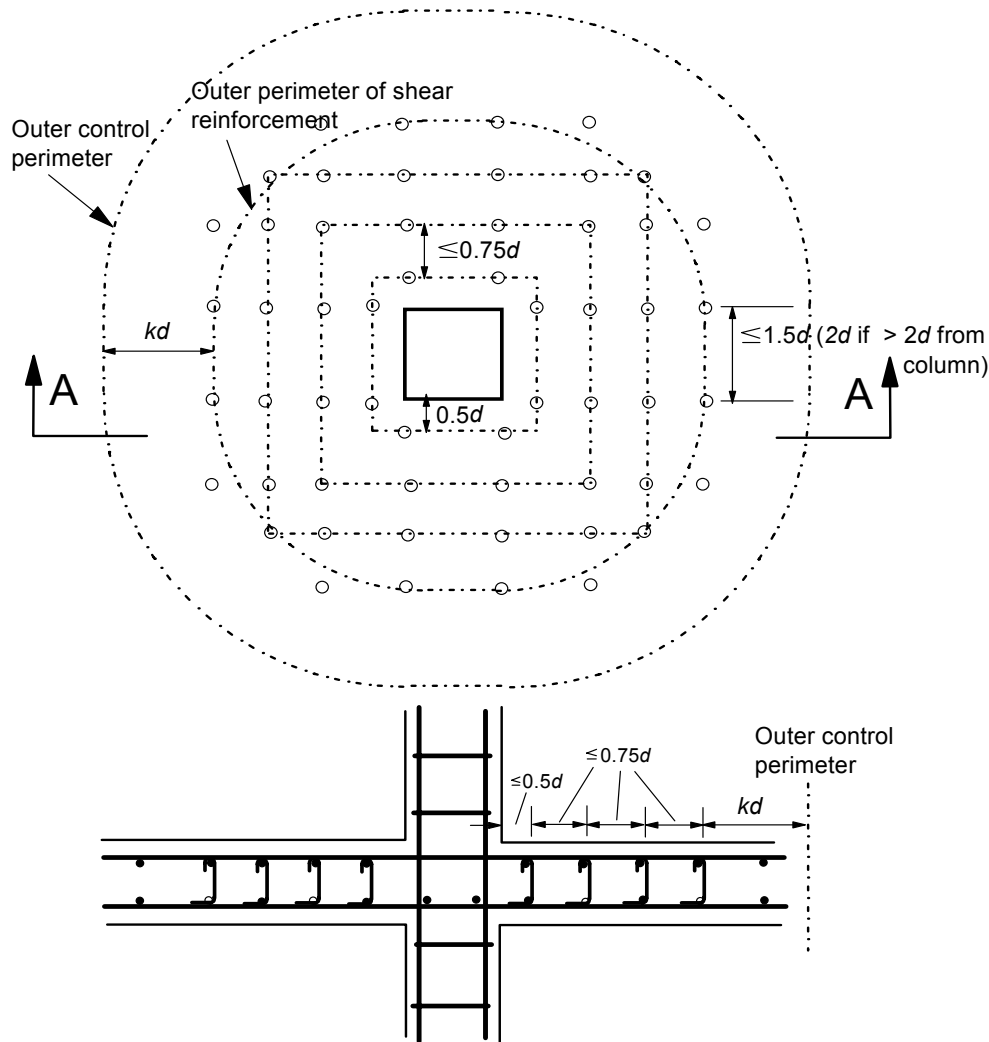
$$V_u = 0,75V_{Rd,c} + V_S$$

Shear reinforcement within $1,5d$ from column is accounted for with

$$f_{y,red} = 250 + 0,25d(mm) \leq f_{ywd}$$



Punching shear reinforcement



The outer control perimeter at which shear reinforcement is not required, should be calculated from:

$$u_{\text{out,ef}} = V_{\text{Ed}} / (v_{\text{Rd,c}} d)$$

The outermost perimeter of shear reinforcement should be placed at a distance not greater than kd ($k = 1.5$) within the outer control perimeter.

Special types of punching shear reinforcement

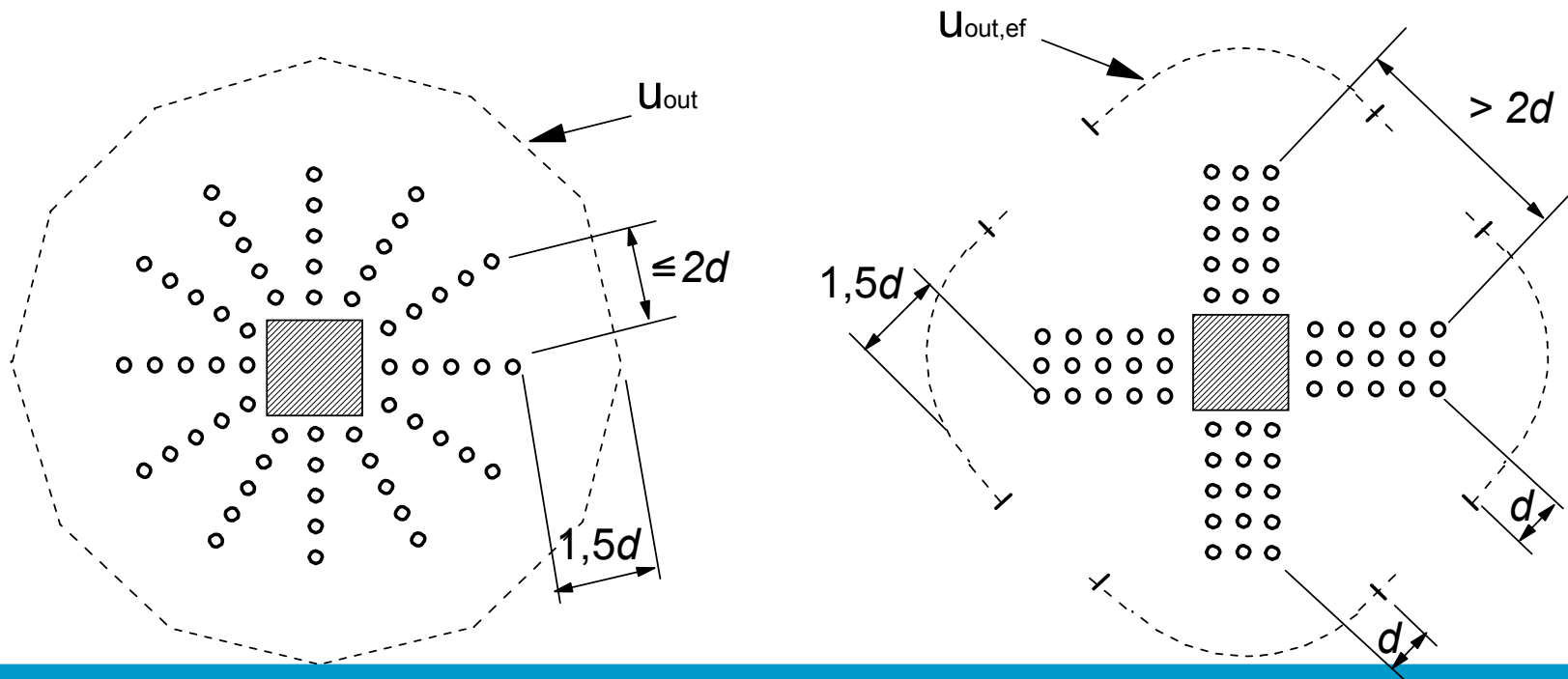
Dowel strips



Punching shear reinforcement

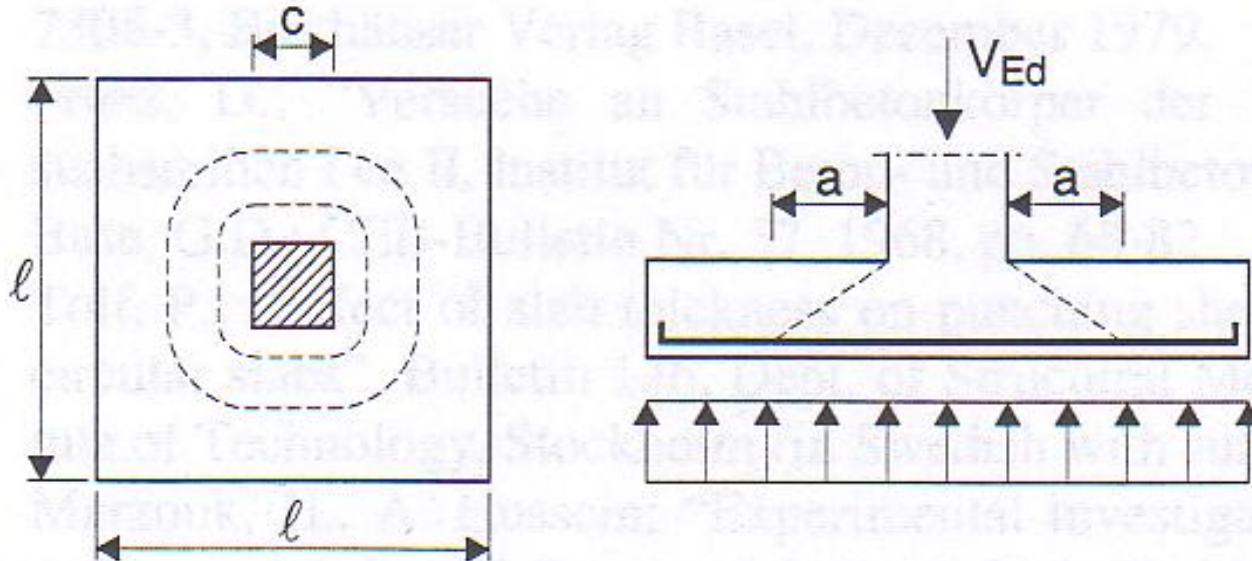
Where proprietary systems are used the control perimeter at which shear reinforcement is not required, u_{out} or $u_{out,ef}$ (see Figure) should be calculated from the following expression:

$$u_{out,ef} = V_{Ed} / (v_{Rd,c} d)$$



Punching shear

- Column bases; critical parameters possible at $a < 2d$
- $V_{Rd} = C_{Rd,c} \cdot k (100 \rho f_{ck})^{1/3} \cdot 2d/a$



Design with strut and tie models

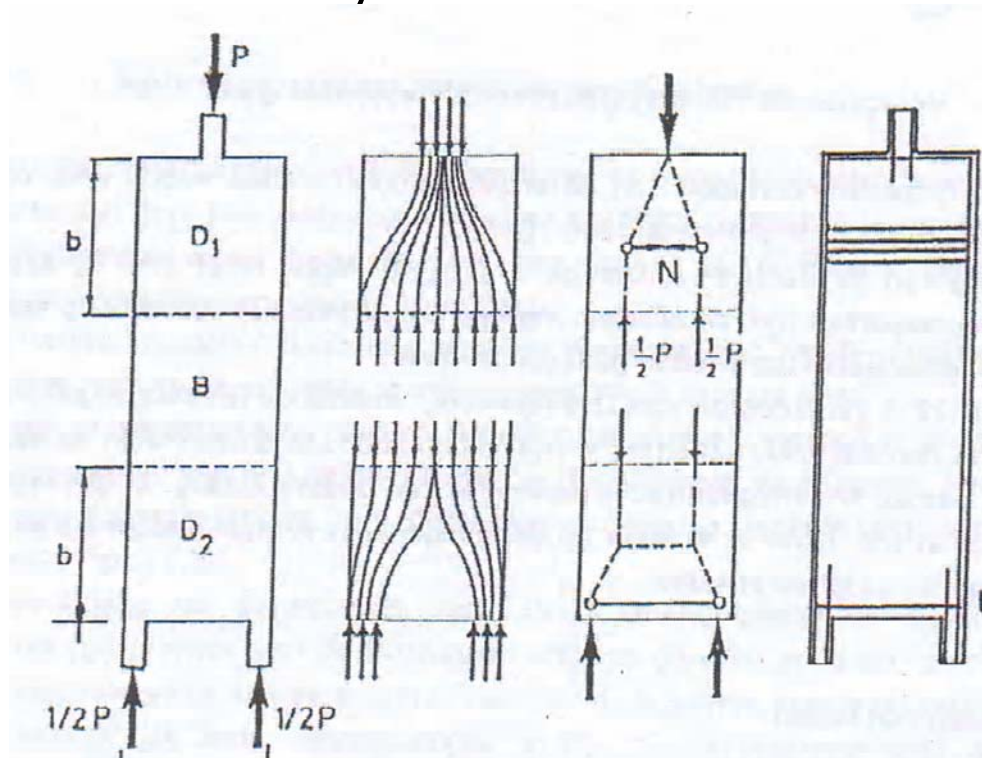
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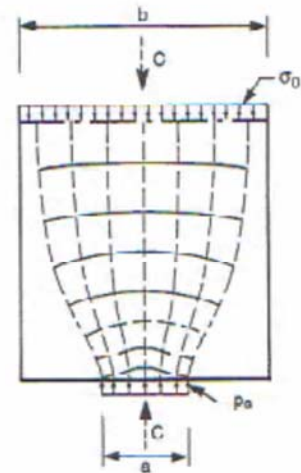
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General idea behind strut and tie models

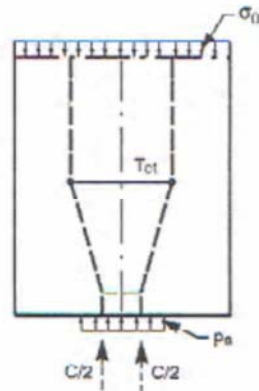
Structures can be subdivided into regions with a steady state of the stresses (B-regions, where "B" stands for "Bernoulli" and in regions with a nonlinear flow of stresses (D-regions, where "D" stands for "Discontinuity")



D-region: stress trajectories and strut and tie model



(a) Stress trajectories

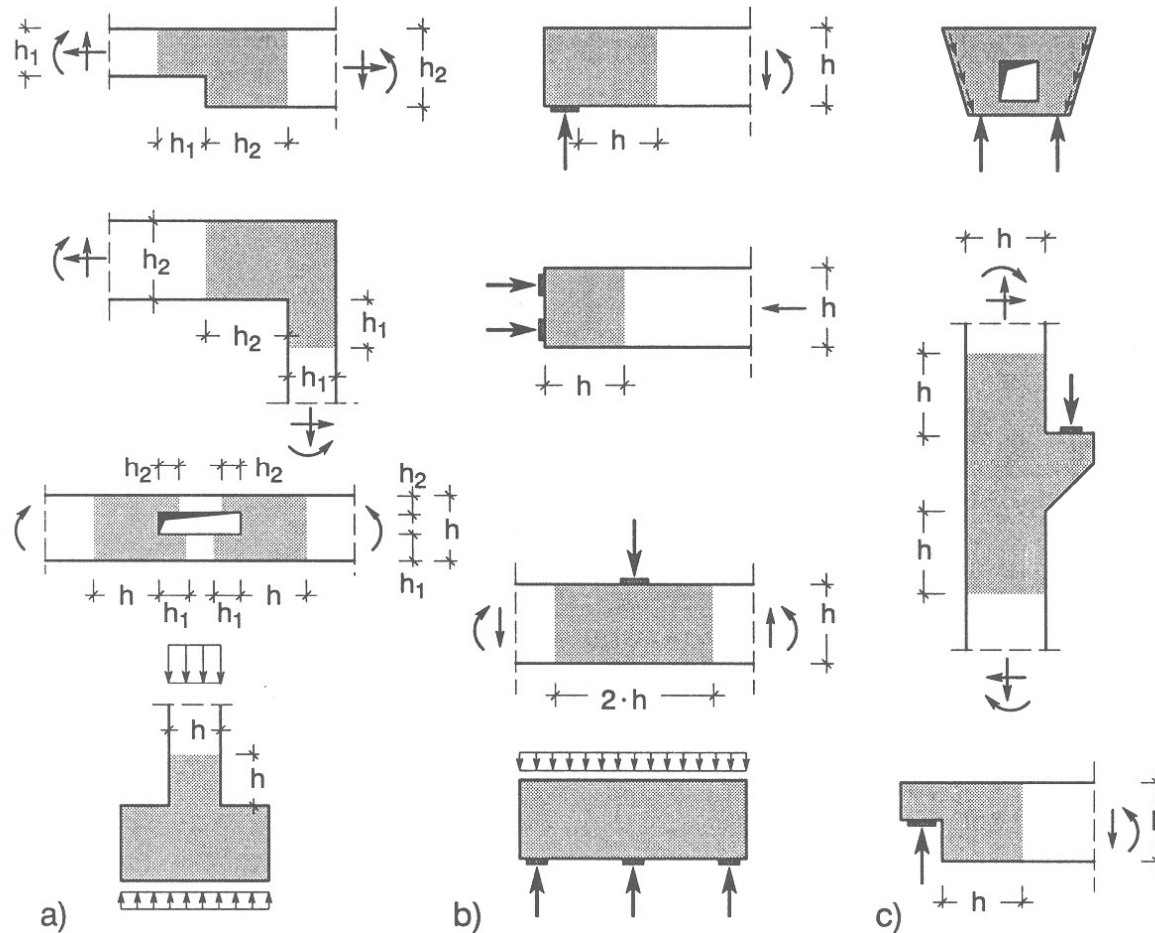


(b) Strut-and-tie model

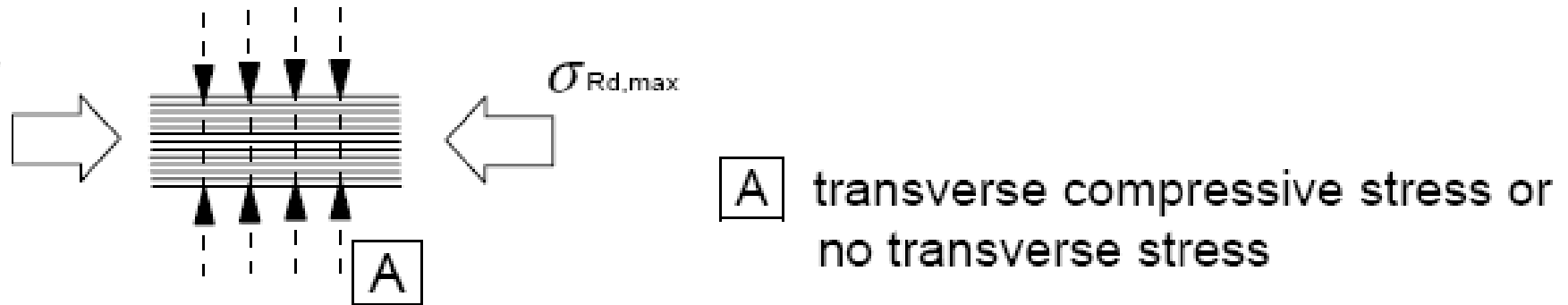
Steps in design:

1. Define geometry of D-region (Length of D-region is equal to maximum width of spread)
2. Sketch stress trajectories
3. Orient struts to compression trajectories
4. Find equilibrium model by adding tensile ties
5. Calculate tie forces
6. Calculate cross section of tie
7. Detail reinforcement

Examples of D-regions in structures



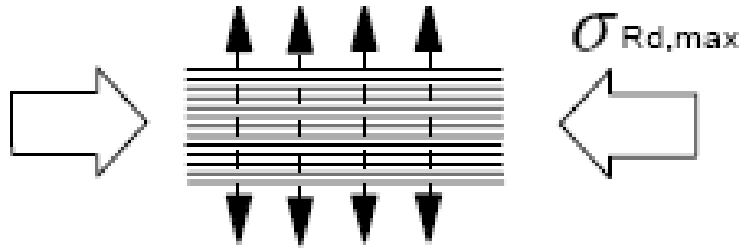
Design of struts, ties and nodes



Struts with transverse compression stress or zero stress:

$$\sigma_{Rd,max} = f_{cd}$$

Design of struts, ties and nodes



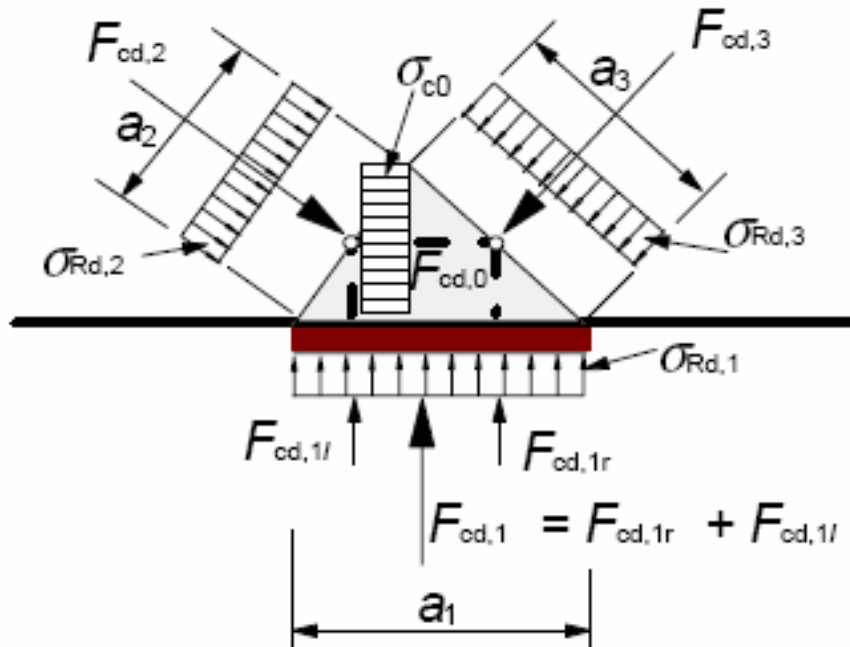
Struts in cracked compression zones, with transverse tension

$$\sigma_{Rd,max} = \nu f_{cd}$$

Recommended value $\nu = 0,60 (1 - f_{ck}/250)$

Design of struts, ties and nodes

Compression nodes without tie



$$\sigma_{Rd,max} = k_1 v' f_{cd}$$

where

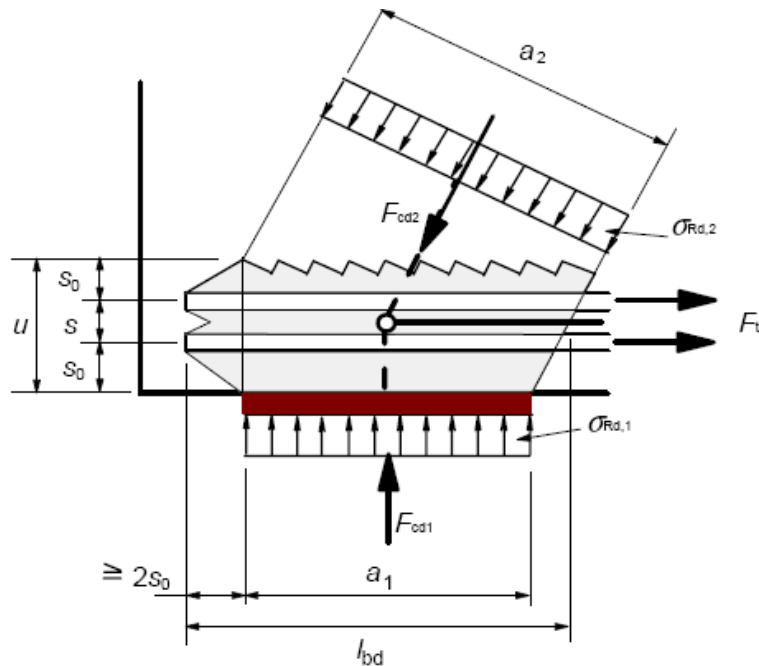
$$v' = 0,60 (1 - f_{ck}/250)$$

Recommended value

$$K_1 = 1,0$$

Design of struts, ties and nodes

Compression-Compression-Tension (CTT) node



$$\sigma_{Rd,max} = k_2 v' f_{cd}$$

where

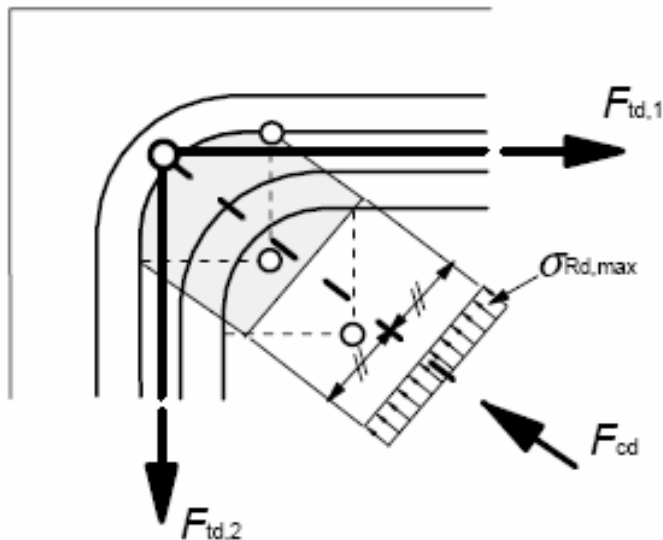
$$v' = 0,60 (1 - f_{ck}/250)$$

Recommended value

$$k_2 = 0,85$$

Design of struts, ties and nodes

Compression-Tension-Tension (CTT) node



$$\sigma_{Rd,max} = k_3 v' f_{cd}$$

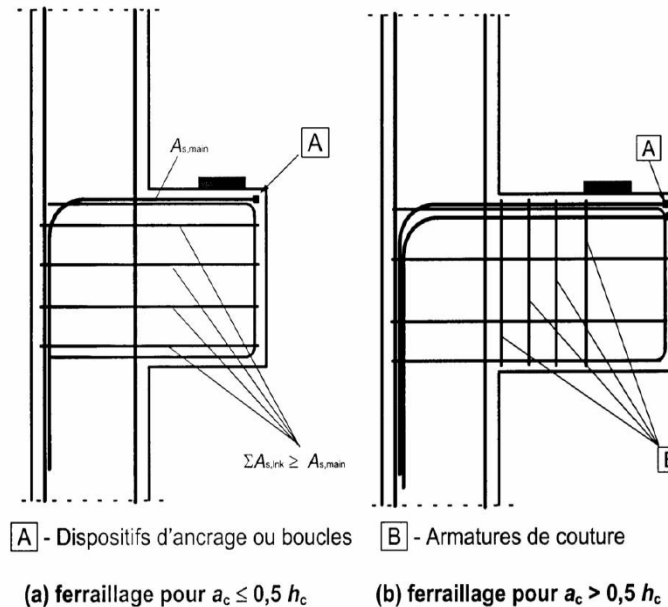
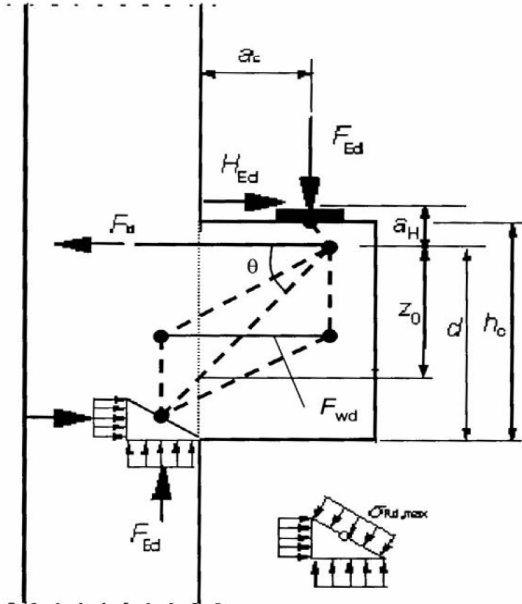
where

$$v' = 0,60 (1 - f_{ck}/250)$$

Recommended value

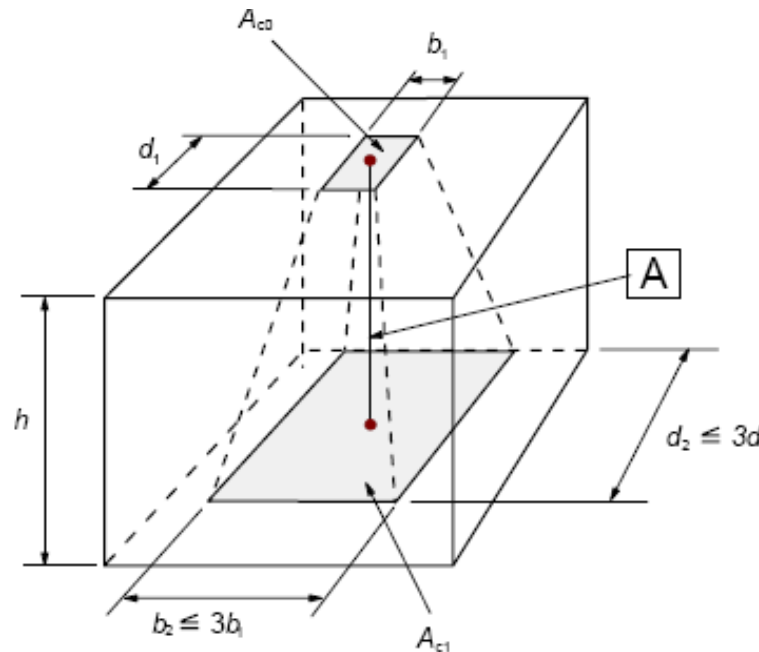
$$k_3 = 0,75$$

Example of detailing based on strut and tie solution



Stress - strain relation for confined concrete (dotted line)

Bearing capacity of partially loaded areas



A - line of action

$$h \geq (b_2 - b_1) \text{ and } \geq (d_2 - d_1)$$

$$F_{Rdu} = A_{c0} \cdot f_{cd} \cdot \sqrt{A_{c1} / A_{c0}} \leq 3,0 \cdot f_{cd} \cdot A_{c0}$$

where:

A_{c0} is the loaded area,

A_{c1} is the maximum design distribution area with a similar shape to A_{c0}

Crack width control in concrete structures

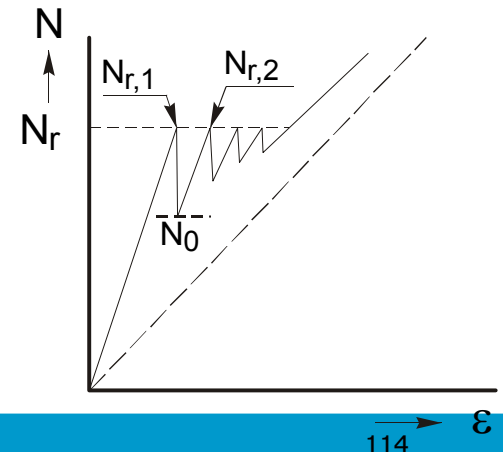
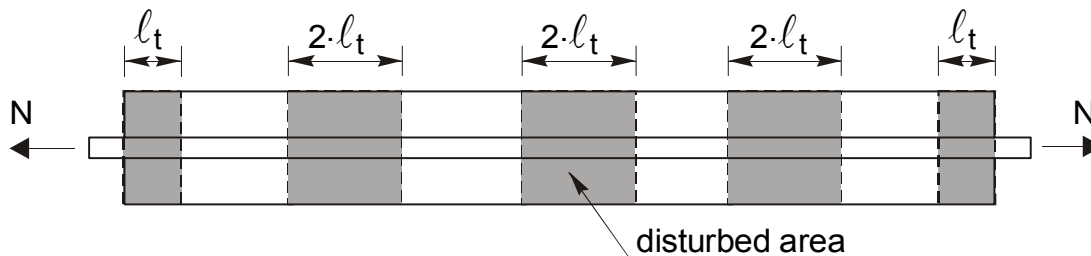
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Theory of crack width control (4)

When more cracks occur, more disturbed regions are found in the concrete tensile bar. In the N - ε relation this stage (the "crack formation stage") is characterized by a "zig-zag"-line ($N_{r,1}$ - $N_{r,2}$). At a certain strain of the bar, the disturbed areas start to overlap. If no intermediate areas are left, the concrete cannot reach the tensile strength anymore, so that no new cracks can occur. The "crack formation stage" is ended and the stabilized cracking stage starts. No new cracks occur, but existing cracks widen.



EC-formulae for crack width control (1)

For the calculation of the maximum (or characteristic) crack width, the difference between steel and concrete deformation has to be calculated for the largest crack distance, which is $s_{r,max} = 2l_t$. So

$$w_k = s_{r,max} (\varepsilon_{sm} - \varepsilon_{cm}) \quad \text{Eq. (7.8)}$$

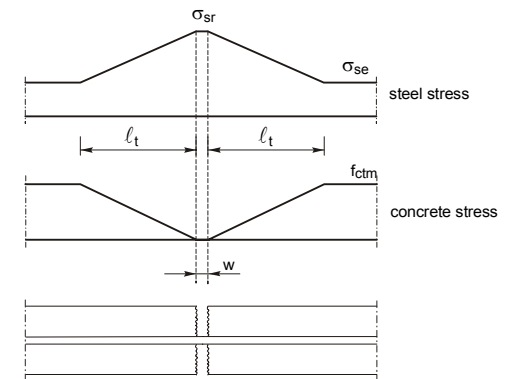
where

$s_{r,max}$ is the maximum crack distance

and

$(\varepsilon_{sm} - \varepsilon_{cm})$ is the difference in deformation between steel and concrete over the maximum crack distance.

Accurate formulations for $s_{r,max}$ and $(\varepsilon_{sm} - \varepsilon_{cm})$ will be given



EC-2 formulae for crack width control (2)

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0,6 \frac{\sigma_s}{E_s} \quad \text{Eq. 7.0}$$

where: σ_s is the stress in the steel assuming a cracked section

α_e is the ratio E_s/E_{cm}

$\rho_{p,eff} = (A_s + \xi A_p)/A_{c,eff}$ (effective reinforcement ratio including eventual prestressing steel A_p)

ξ is bond factor for prestressing strands or wires

k_t is a factor depending on the duration of loading (0,6 for short and 0,4 for long term loading)

EC-3 formulae for crack width control (4)

Maximum final crack spacing $s_{r,max}$

$$s_{r,max} = 3.4c + 0.425 k_1 k_2 \frac{\phi}{\rho_{p,eff}} \quad (\text{Eq. 7.11})$$

where c is the concrete cover

ϕ is the bar diameter

k_1 bond factor (0,8 for high bond bars, 1,6 for bars with an effectively plain surface (e.g. prestressing tendons))

k_2 strain distribution coefficient (1,0 for tension and 0,5 for bending: intermediate values can be used)

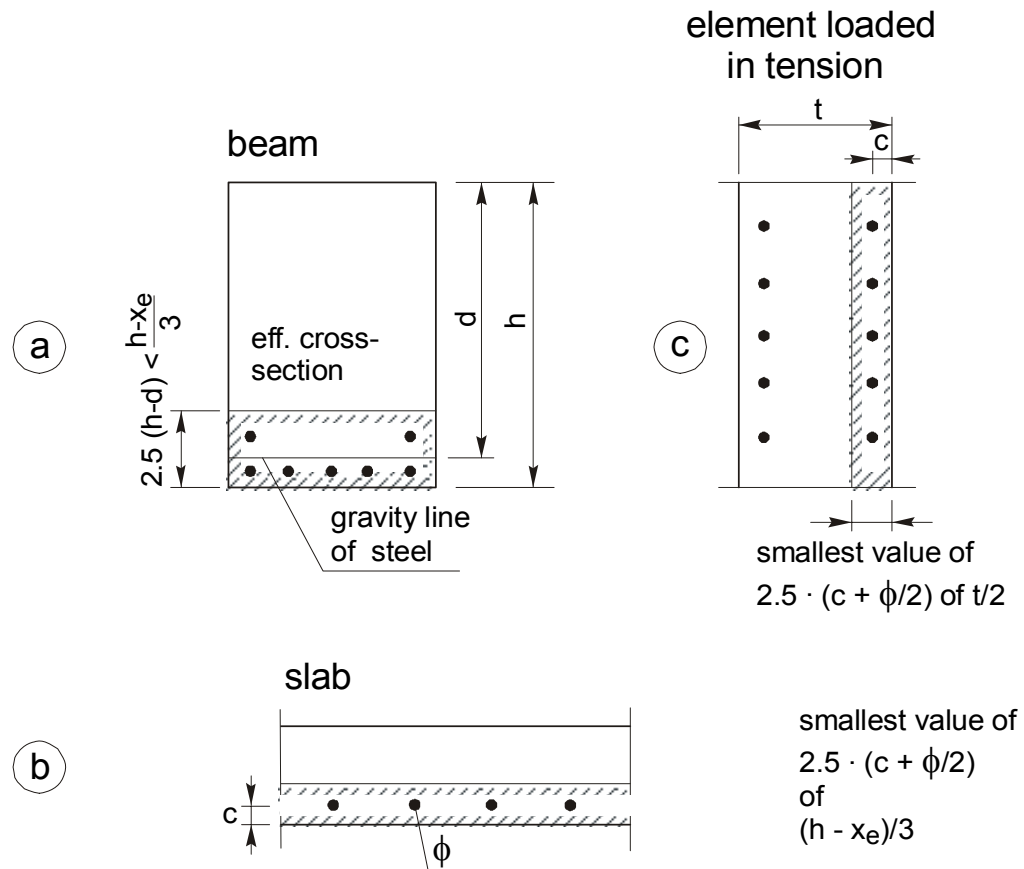
EC-2 requirements for crack width control (recommended values)

Exposure class	RC or unbonded PSC members	Prestressed members with bonded tendons
	Quasi-permanent load	Frequent load
X0,XC1	0.3	0.2
XC2,XC3,XC4	0.3	
XD1,XD2,XS1,XS2, XS3		Decompression

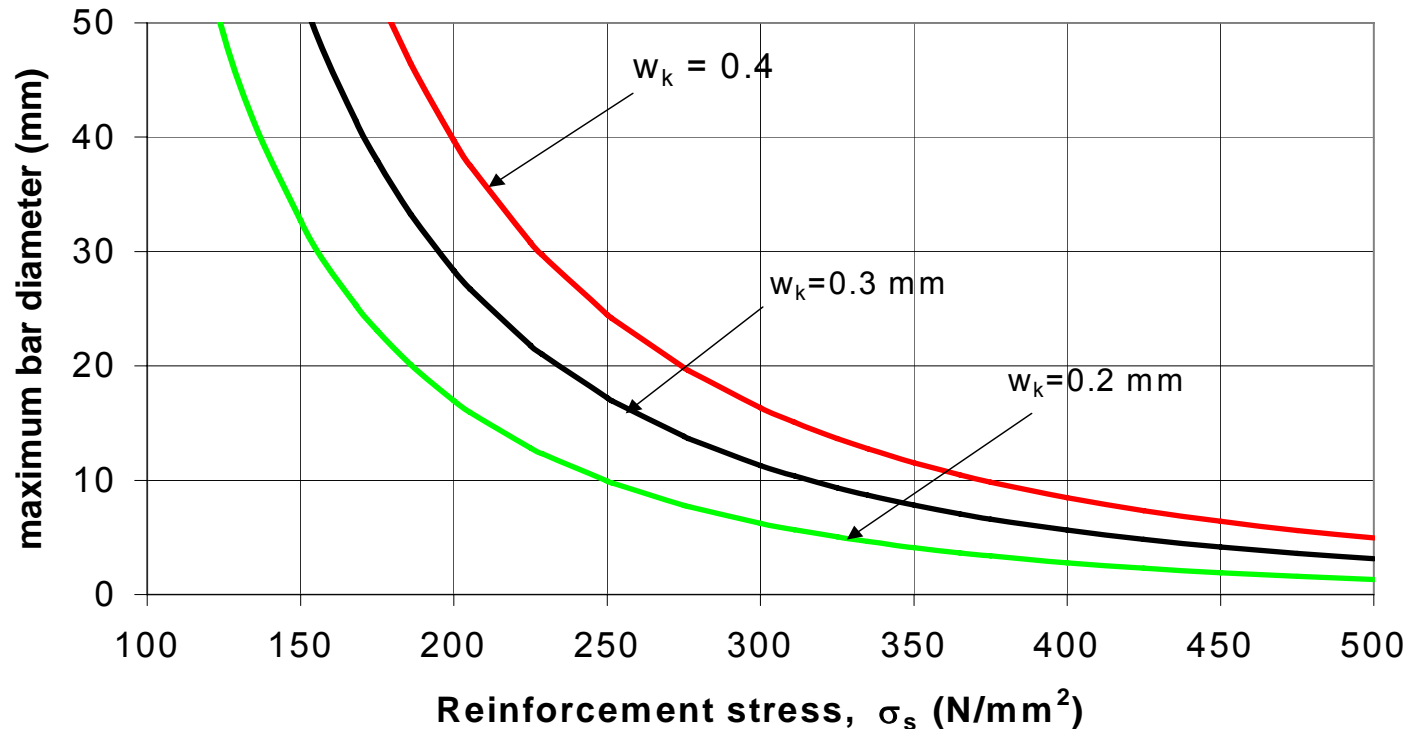
EC-2 formulae for crack width control (5)

In order to be able to apply the crack width formulae, basically valid for a concrete tensile bar, to a structure loaded in bending, a definition of the “effective tensile bar height” is necessary. The effective height $h_{c,ef}$ is the minimum of:

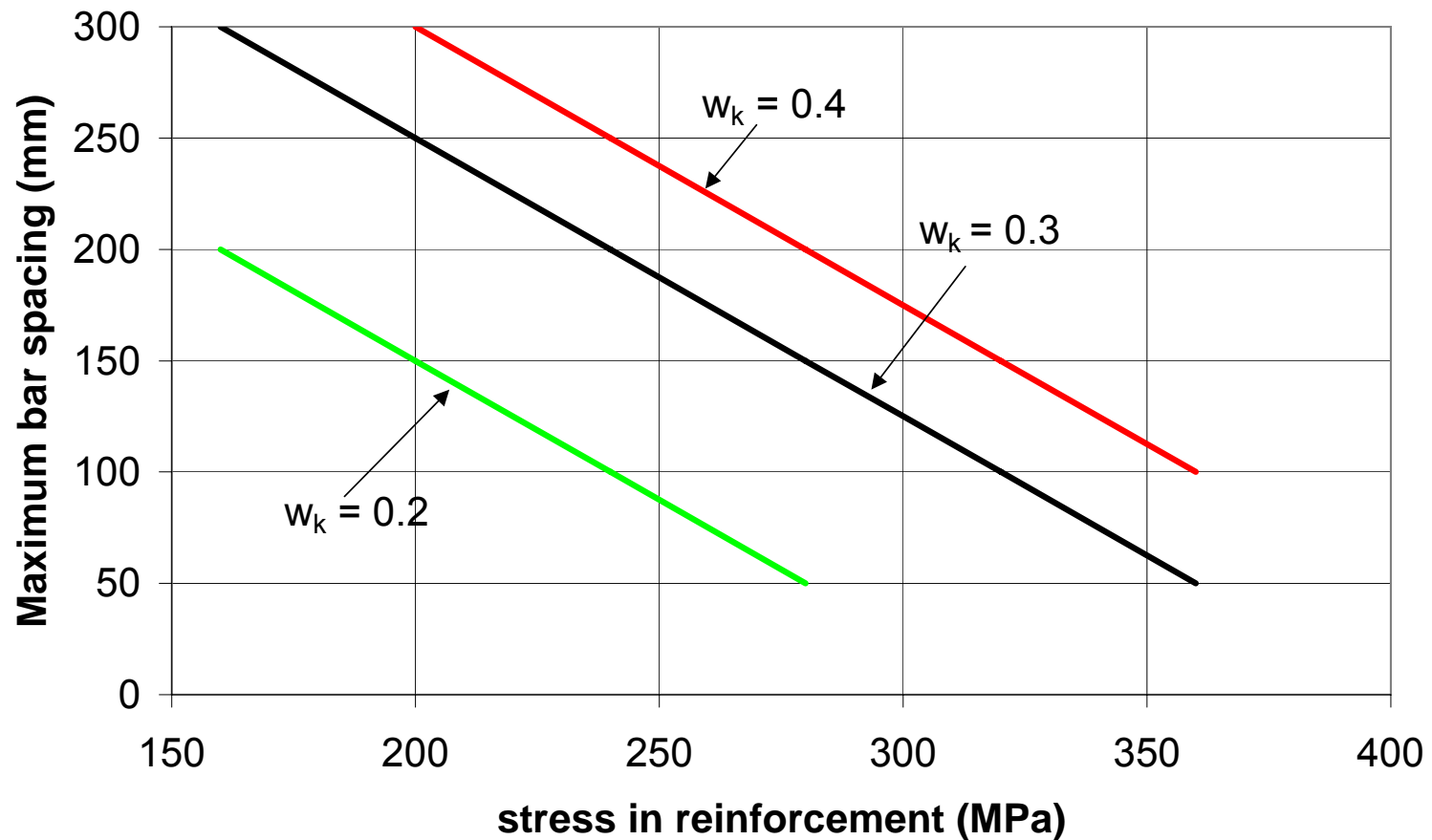
$$\begin{aligned} &2,5 (h-d) \\ &(h-x)/3 \\ &h/2 \end{aligned}$$



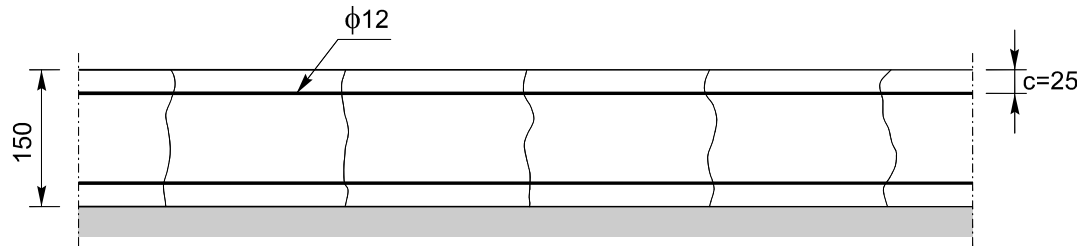
Maximum bar diameters for crack control (simplified approach 7.3.3)



Maximum bar spacing for crack control (simplified approach 7.3.3)



Example (1)



Continuous concrete road

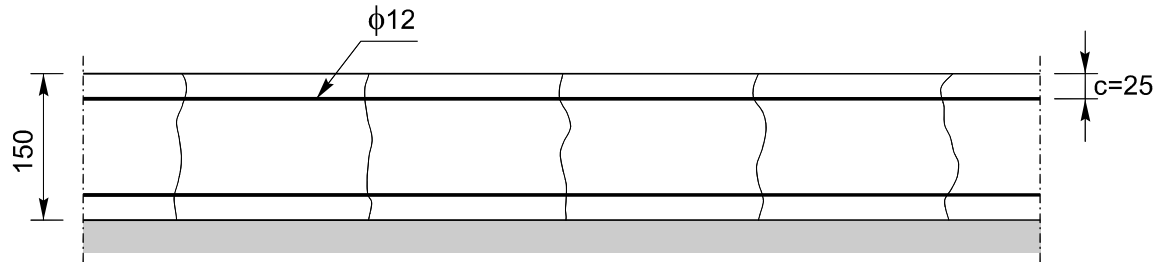
Data: Concrete C20/25, $f_{ctm} = 2,2 \text{ MPa}$, shrinkage $\varepsilon_{sh} = 0,25 \cdot 10^{-3}$, temperature difference in relation to construction situation $\Delta T = 25^\circ$. Max. crack width allowed = 0,2mm.

Calculation

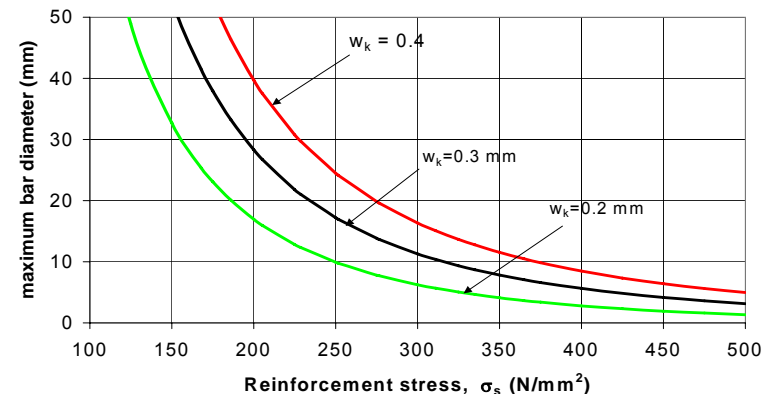
The maximum imposed deformation (shrinkage + temperature) is $\varepsilon_{tot} = 0,50 \cdot 10^{-3}$. Loading is slow, so $E_{c,\infty} = E_c / (1 + \varphi) \cong 30.000 / (1 + 2) = 10.000 \text{ MPa}$. At $\varepsilon_{tot} = 0,50 \cdot 10^{-3}$ a concrete tensile strength of 5 MPa applies, so the road is cracked.

Cont. →

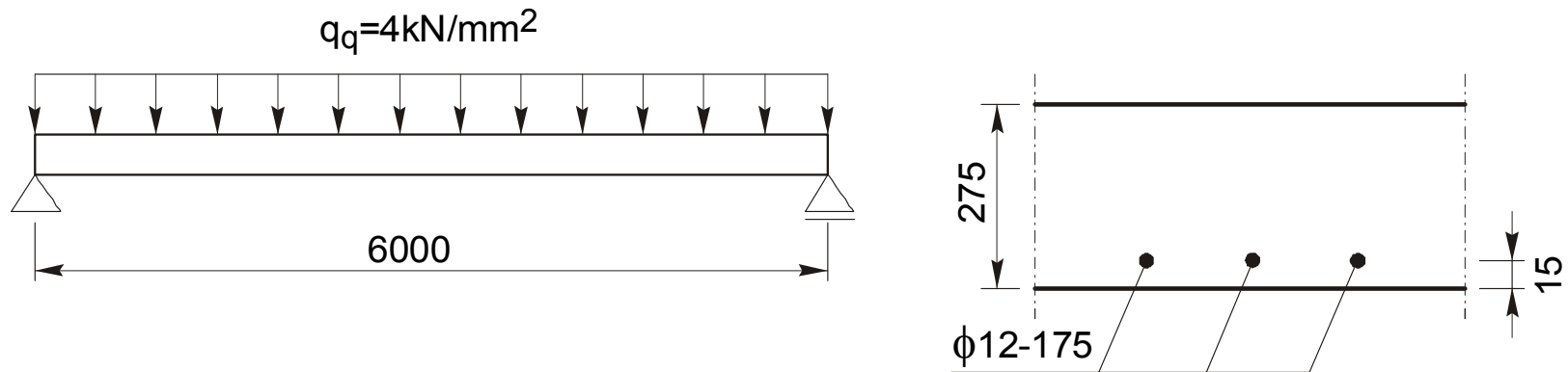
Example (1, cont.)



For imposed deformation the “crack formation stage” applies. So, the load will not exceed the cracking load, which is $N_{cr} = A_c(1+n\rho)f_{ctm} \cong 1,1A_c f_{ctm} = 330 \text{ kN}$ for $b = 1\text{m}$. From the diagram at the right it is found that a diameter of 12mm would require a steel stress not larger than 225 MPa. To meet this requirement $d = 12\text{mm}$ bars at distances 150mm, both at top and bottom, are required.

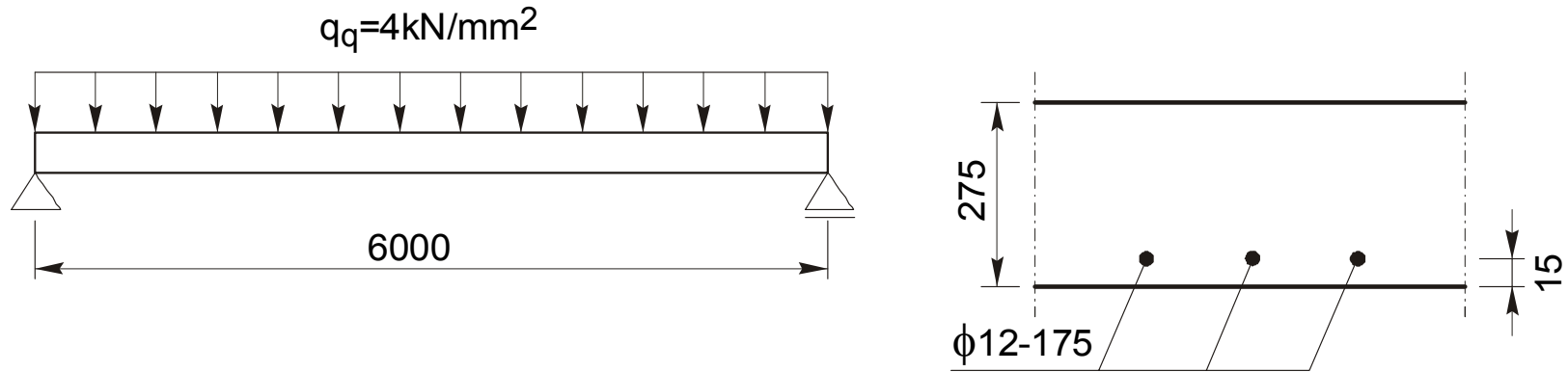


Example (2)



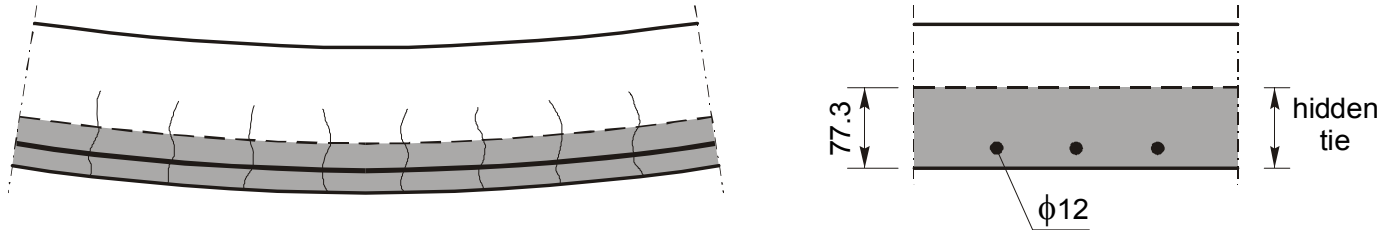
A slab bearing into one direction is subjected to a maximum variable load of 4 kN/m^2 . It should be demonstrated that the maximum crack width under the quasi permanent load combination is not larger than 0,4mm. (The floor is a part of a shopping centre: the environmental class is X0) (cont.→)

Example (2)



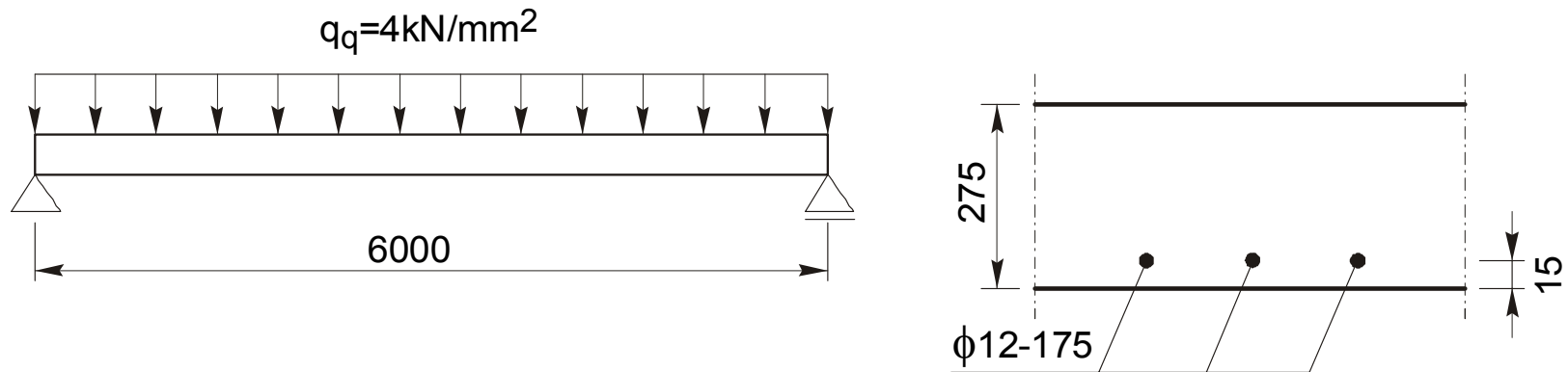
The governing load for the quasi-permanent load combination is:
 $q = q_g + \psi_2 \cdot q_{\text{var.}} = (0.275 \cdot 2500) + 0,6 \cdot 400 = 928 \text{ kg/m}^2$. The maximum bending moment is then $M = 9,28 \cdot 6^2 / 8 = 41,8 \text{ kNm/m'}$. For this bending moment the stress in the steel is calculated as $\sigma_s = 289 \text{ MPa}$. Cont.→

Example (2)



The effective height of the tensile tie is the minimum of $2,5(c + \phi/2)$ or $(h-x)/3$, where x = height of compression zone, calculated as 44mm. So, the governing value is $(h-x)/3 = 77$ mm. The effective reinforcement ratio is then $\rho_{\text{eff}} = (113/0,175)/(77 \cdot 1000) = 0,83 \cdot 10^{-2}$. The crack distance $s_{r,\text{amx}}$ (Eq. 7.11) is found to be 245mm. For the term $(\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}})$ a value $1,0 \cdot 10^{-3}$ is found. This leads to a cracks width equal to $w_k = 0,25$ mm, which is smaller than the required 0,4mm.

Example (2)



A slab bearing into one direction is subjected to a maximum variable load of 4 kN/m^2 . It should be demonstrated that the maximum crack width under the quasi permanent load combination is not larger than 0,4mm. (The floor is a part of a shopping centre: the environmental class is X0) (cont.→)

Deformation of concrete structures

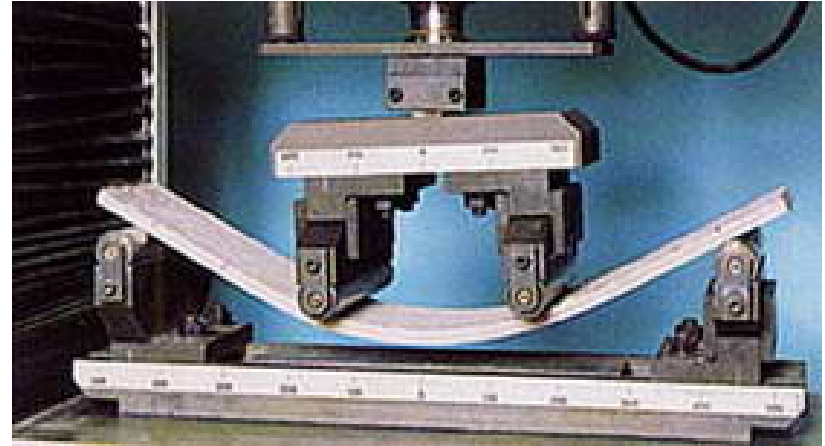
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Deformation of concrete

Reason to worry or
challenge for the future?



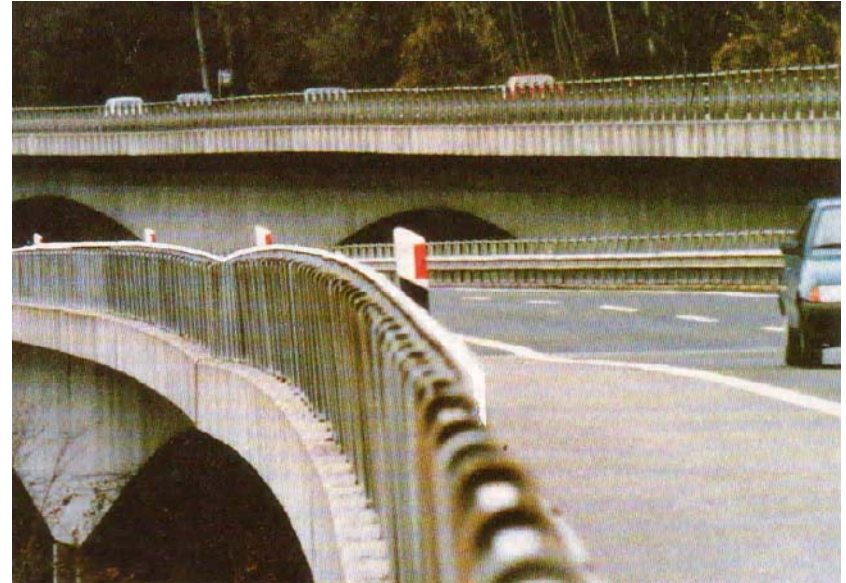
Deflection of ECC specimen, V. Li, University of Michigan

Damage in masonry wall due to excessive
deflection of lintel

Reasons for controlling deflections (1)

Appearance

Deflections of such a magnitude that members appear visibly to sag will upset the owners or occupiers of structures. It is generally accepted that a deflection larger than $\text{span}/250$ should be avoided from the appearance point of view. A survey of structures in Germany that had given rise to complaints

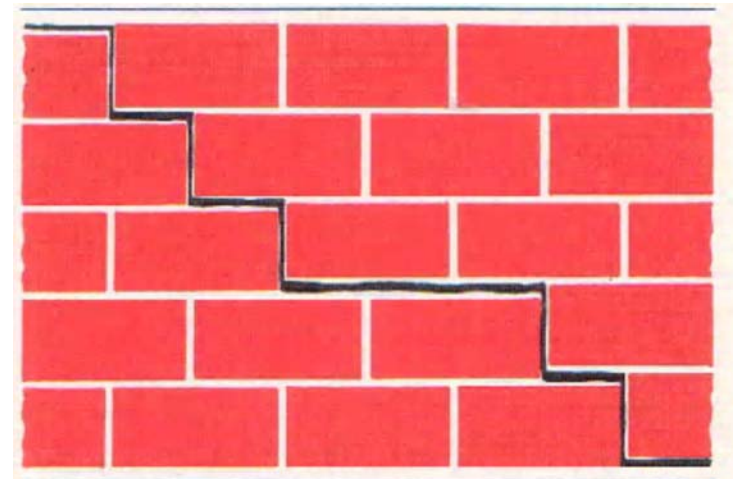
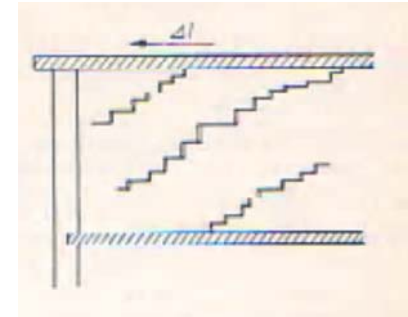


produced 50 examples. The measured sag was less than $\text{span}/250$ in only two of these.

Reasons for controlling deflections (2)

Damage to non-structural Members

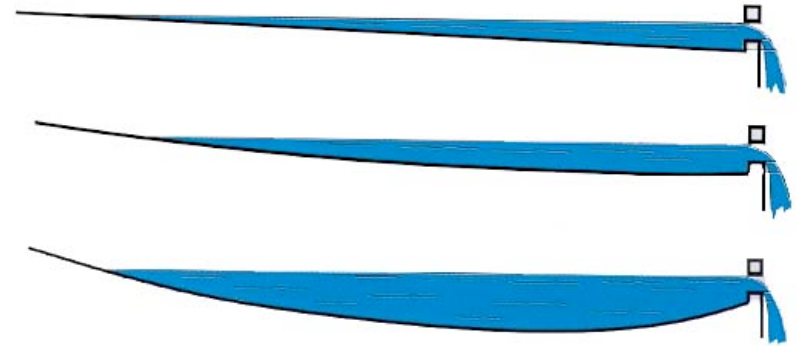
An important consequence of excessive deformation is damage to non structural members, like partition walls. Since partition walls are unreinforced and brittle, cracks can be large (several millimeters). The most commonly specified limit deflection is $\text{span}/500$, for deflection occurring after construction of the partitions. It should be assumed that all quasi permanent loading starts at the same time.



Reasons for controlling deflections (3)

Collapse

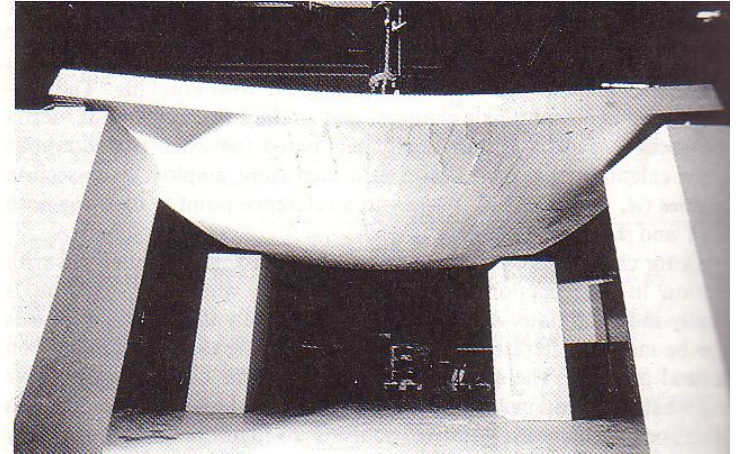
In recent years many cases of collapse of flat roofs have been noted. If the rainwater pipes have a too low capacity, often caused by pollution and finally stoppage, the roof deflects more and more under the weight of the water and finally collapses. This occurs predominantly with light roofs. Concrete roofs are less susceptible for this type of damage



EC-2 Control of deflections

Deflection limits according to chapter 7.4.1

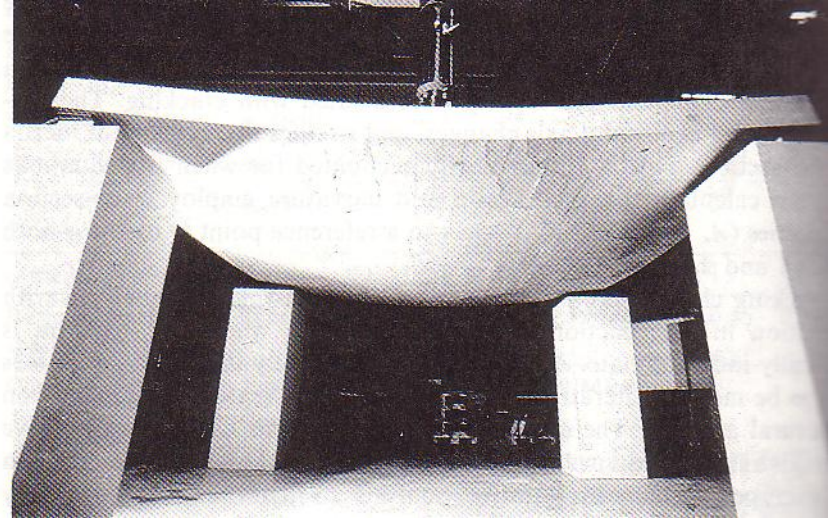
- Under the quasi permanent load the deflection should not exceed $\text{span}/250$, in order to avoid impairment of appearance and general utility
- Under the quasi permanent loads the deflection should be limited to $\text{span}/500$ after construction to avoid damage to adjacent parts of the structure



EC-2: SLS - Control of deflections

Control of deflection can be done in two ways

- By calculation
- By tabulated values



Calculating the deflection of a concrete member

The deflection follows from:

$$\delta = \zeta \delta_{II} + (1 - \zeta) \delta_I$$

δ deflection

δ_I deflection fully cracked

δ_{II} deflection uncracked

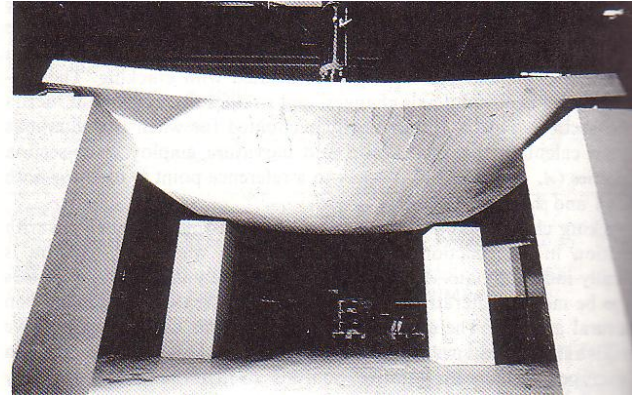
ζ coefficient for tension stiffening (transition coefficient)

$$\zeta = 1 - \beta (\sigma_{sr}/\sigma_s)^2$$

σ_{sr} steel stress at first cracking

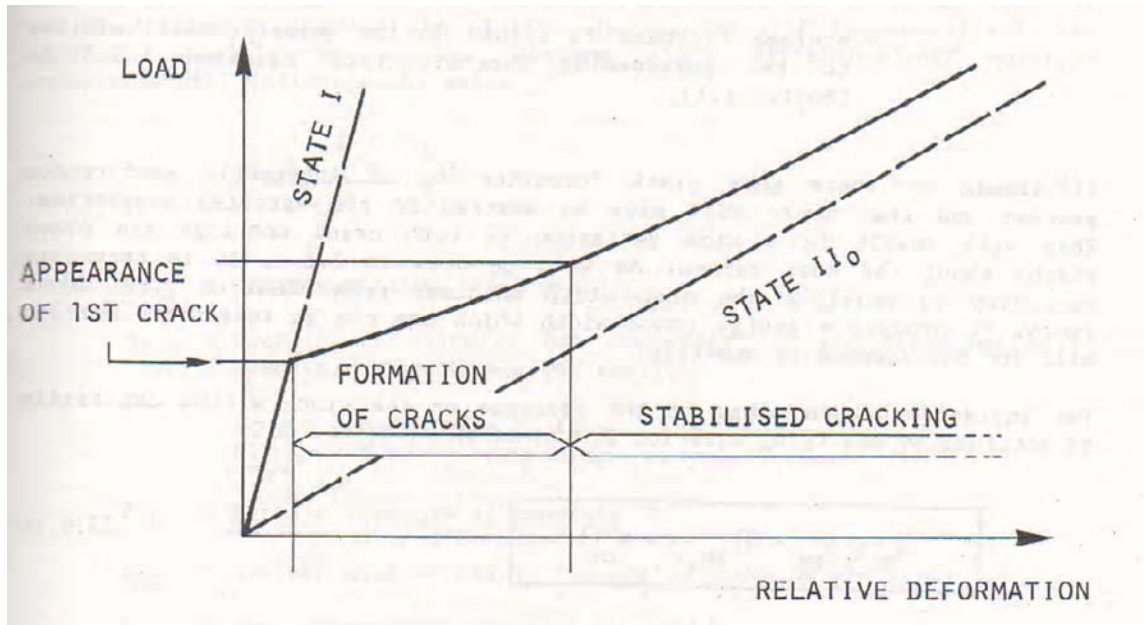
σ_s steel stress at quasi permanent service load

β 1,0 for single short-term loading
0,5 for sustained loads or repeated loading



Calculating the deflection of a concrete member

The transition from the uncracked state (I) to the cracked state (II) does not occur abruptly, but gradually. From the appearance of the first crack, realistically, a parabolic curve can be followed which approaches the line for the cracked state (II).



Calculating the deflection of a concrete member

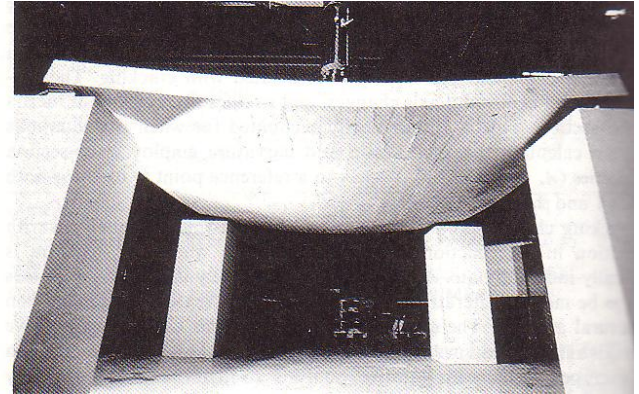
For pure bending the transition factor

$$\xi = 1 - \beta(\sigma_s / \sigma_r)^2$$

can as well be written as

$$\xi = 1 - \beta(M_{cr} / M)^2$$

where M_{cr} is the cracking moment and M is the applied moment



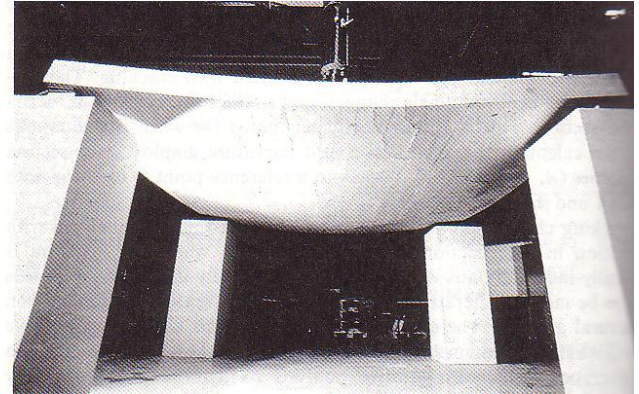
Calculating the deflection of a concrete member

7.4.3 (7)

“The most rigorous method of assessing deflections using the method given before is to compute the curvatures at frequent locations along the member and then calculate the deflection by numerical integration.

In most cases it will be acceptable to compute the deflection twice, assuming the whole member to be in the uncracked and fully cracked condition in turn, and then interpolate using the expression:

$$\xi = 1 - \beta(M_{cr} / M)^2$$

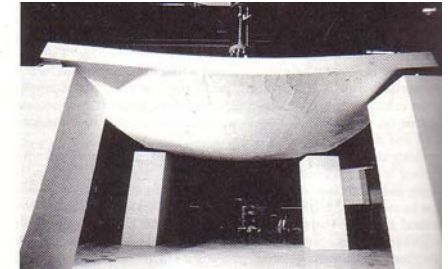


Cases where detailed calculation may be omitted

In order to simplify the design, expressions have been derived, giving limits of l/d for which no detailed calculation of the deflection has to be carried out.

These expressions are the results of an extended parameter analysis with the method of deflection calculation as given before. The slenderness limits have been determined with the criteria $\delta < L/250$ for quasi permanent loads and $\delta < L/500$ for the additional load after removing the formwork

The expressions, which will be given at the next sheet, have been calculated for an assumed steel stress of 310 MPa at midspan of the member. Where other stress levels are used, the values obtained by the expressions should be multiplied with $310/\sigma_s$



Calculating the deflection of a concrete member

For span-depth ratios below the following limits no further checks is needed

$$\frac{l}{d} = K \left[11 + 1,5 \sqrt{f_{ck}} \frac{\rho_0}{\rho} + 3,2 \sqrt{f_{ck}} \left(\frac{\rho_0}{\rho} - 1 \right)^{3/2} \right] \quad \text{if } \rho \leq \rho_0 \quad (7.16.a)$$

$$\frac{l}{d} = K \left[11 + 1,5 \sqrt{f_{ck}} \frac{\rho_0}{\rho - \rho'} + \frac{1}{12} \sqrt{f_{ck}} \sqrt{\frac{\rho'}{\rho_0}} \right] \quad \text{if } \rho > \rho_0 \quad (7.16.b)$$

l/d is the limit span/depth

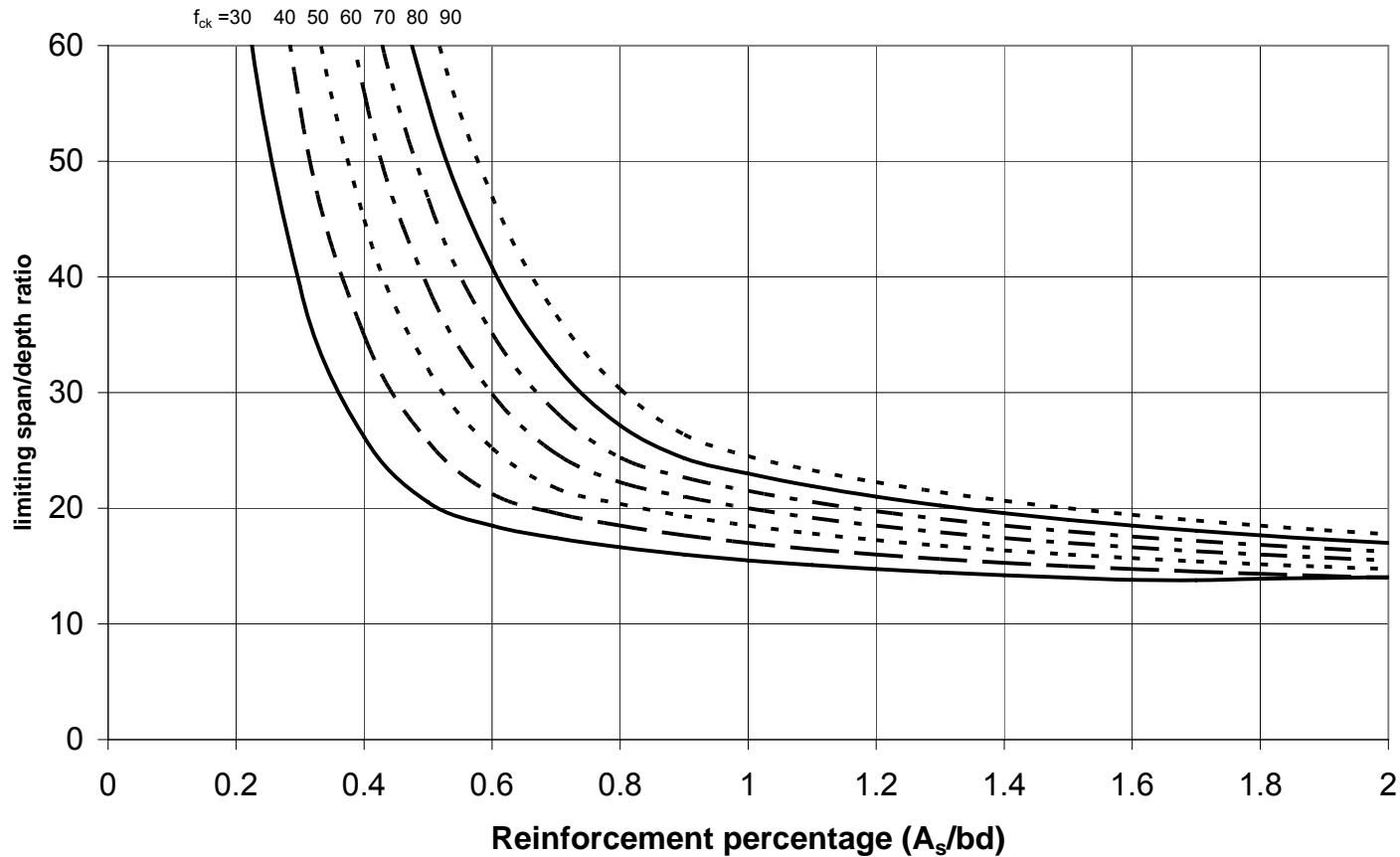
K is the factor to take into account the different structural systems

ρ_0 is the reference reinforcement ratio = $\sqrt{f_{ck}} \cdot 10^{-3}$

ρ is the required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers)

ρ' is the required compression reinforcement ratio at mid-span to resist the moment due to design loads (at support for cantilevers)

Previous expressions in a graphical form (Eq. 7.16):



Limit values for l/d below which no calculated verification of the deflection is necessary

The table below gives the values of K (Eq.7.16), corresponding to the structural system. The table furthermore gives limit l/d values for a relatively high ($\rho=1,5\%$) and low ($\rho=0,5\%$) longitudinal reinforcement ratio. These values are calculated for concrete C30 and $\sigma_s = 310$ MPa and satisfy the deflection limits given in 7.4.1 (4) and (5).

Structural system	K	$\rho = 0,5\%$	$\rho = 1,5\%$
Simply supported slab/beam	1,0	$l/d=14$	$l/d=20$
End span	1,3	$l/d=18$	$l/d=26$
Interior span	1,5	$l/d=20$	$l/d=30$
Flat slab	1,2	$l/d=17$	$l/d=24$
Cantilever	0,4	$l/d= 6$	$l/d=8$

Bond and anchorage

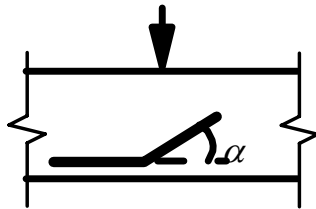
Ultimate Bond Stress, f_{bd} (8.4.2)

- The design value of the ultimate bond stress, $f_{bd} = 2,25 \eta_1 \eta_2 f_{ctd}$ where f_{ctd} should be limited to C60/75

$\eta_1 = 1$ for 'good' and 0,7 for 'poor' bond conditions

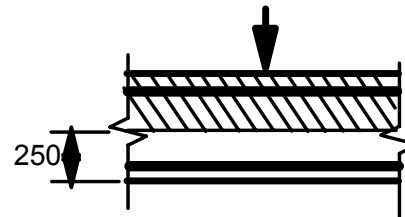
$\eta_2 = 1$ for $\phi \leq 32$, otherwise $(132 - \phi)/100$

Direction of concreting



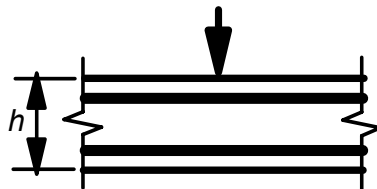
a) $45^\circ \leq \alpha \leq 90^\circ$

Direction of concreting



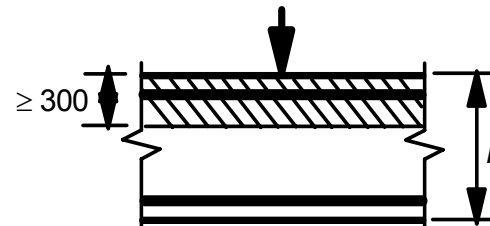
c) $h > 250$ mm

Direction of concreting



b) $h \leq 250$ mm

Direction of concreting



d) $h > 600$ mm

a) & b) 'good' bond conditions for all bars

c) & d) unhatched zone – 'good' bond conditions
hatched zone - 'poor' bond conditions

Basic Required Anchorage Length, $l_{b,rqd}$

(8.4.3)

$$l_{b,rqd} = (\phi / 4) (\sigma_{sd} / f_{bd})$$

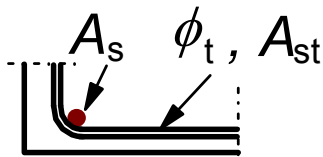
where σ_{sd} is the design stress of the bar at the position from where the anchorage is measured

- For bent bars $l_{b,rqd}$ should be measured along the centreline of the bar
- Where pairs of wires/bars form welded fabrics ϕ should be replaced by $\phi_n = \phi\sqrt{2}$

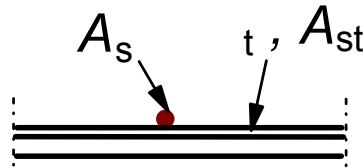
Design Anchorage Length, l_{bd} (8.4.4)

$$l_{bd} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 l_{b,rqd} \geq l_{b,min}$$

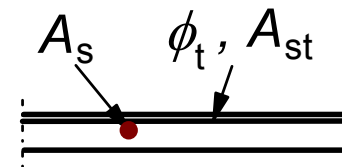
- α_1 effect of bends For straight bars $\alpha_1 = 1.0$, otherwise 0.7
- α_2 effect of concrete cover $\alpha_2 = 1 - 0.15(\text{cover} - \phi)/\phi \geq 0.7$ and ≤ 1.0
- α_3 effect of confinement by transverse reinforcement (not welded)
- $\alpha_3 = 1 - K\lambda \geq 0.7$ and ≤ 1.0 where $\lambda = (\Sigma A_{st} - \Sigma A_{st,min})/A_s$



$$K = 0.1$$



$$K = 0.05$$



$$K = 0$$

- α_4 effect of confinement by welded transverse reinforcement $\alpha_4 = 0.7$

- α_5 effect of confinement by transverse pressure

$$\alpha_5 = 1 - 0.04p \geq 0.7 \text{ and } \leq 1.0$$

where p is the transverse pressure (MPa) at ULS along l_{bd}

$$(\alpha_2 \alpha_3 \alpha_5) \geq 0.7$$

$$l_{b,min} > \max(0.3l_b; 15\phi, 100\text{mm})$$

Design Lap Length, l_0 (8.7.3)

$$l_0 = \alpha_1 \alpha_2 \alpha_3 \alpha_5 \alpha_6 l_{b,rqd} \geq l_{0,min}$$

$\alpha_1 \alpha_2 \alpha_3 \alpha_5$ are as defined for anchorage length

$$\alpha_6 = (\rho_1/25)^{0,5} \text{ but between } 1,0 \text{ and } 1,5$$

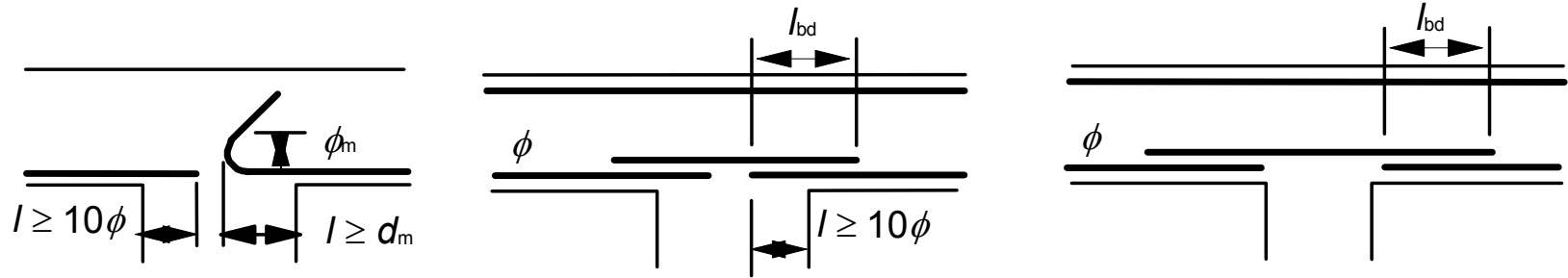
where ρ_1 is the % of reinforcement lapped within $0,65l_0$ from the centre of the lap

Percentage of lapped bars relative to the total cross-section area	< 25%	33%	50%	>50%
α_6	1	1,15	1,4	1,5
Note: Intermediate values may be determined by interpolation.				

$$l_{0,min} \geq \max\{0,3 \alpha_6 l_{b,rqd}; 15\phi; 200\}$$

Anchorage of Bottom Reinforcement at Intermediate Supports

(9.2.1.5)



- Anchorage length, $l, \geq 10\phi$ for straight bars
 $\geq \phi_m$ for hooks and bends with $\phi \geq 16\text{mm}$
 $\geq 2\phi_m$ for hooks and bends with $\phi < 16\text{mm}$
- Continuity through the support may be required for robustness (Job specification)

Supporting Reinforcement at 'Indirect' Supports

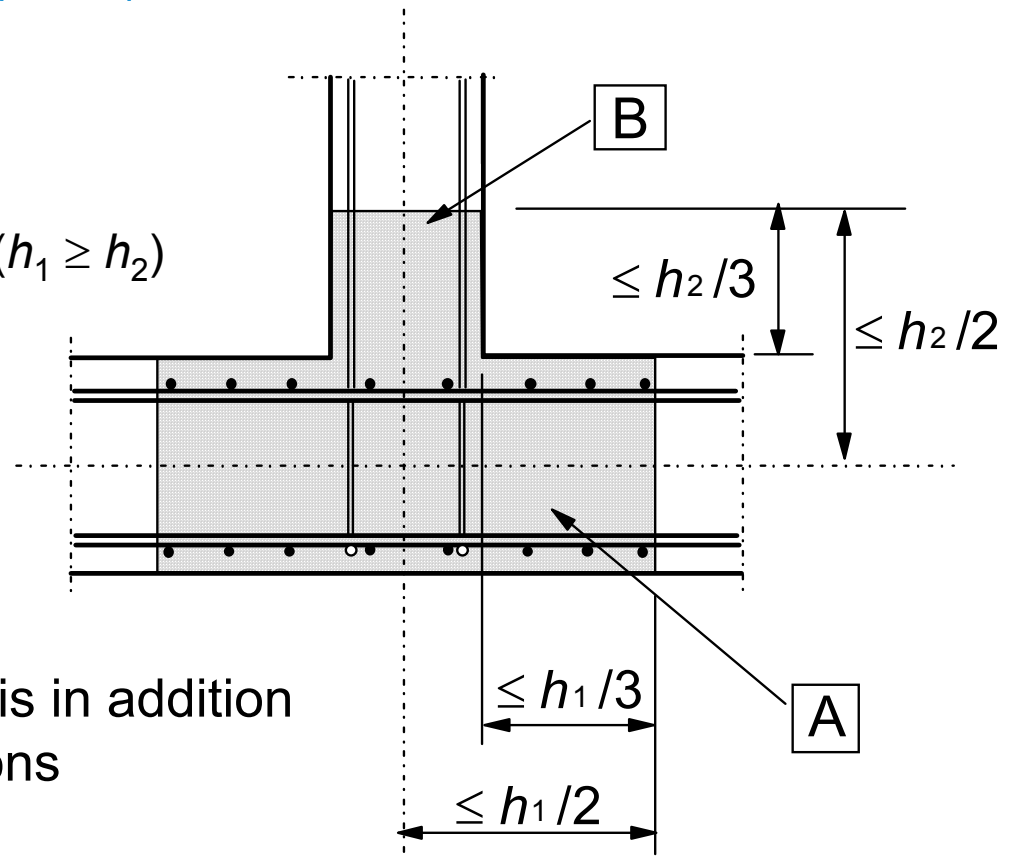
(9.2.5)

A

supporting beam with height h_1

B

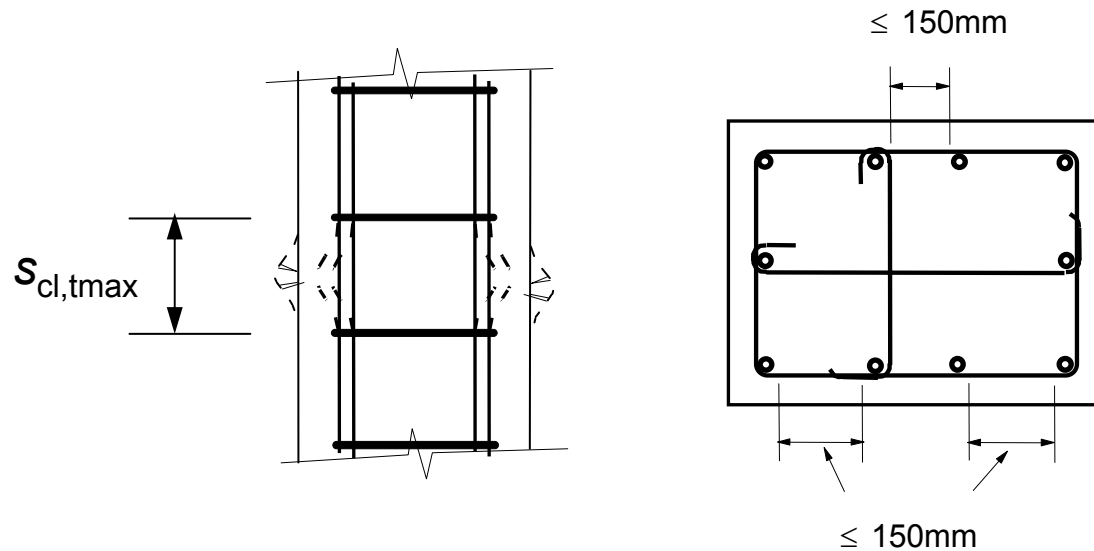
supported beam with height h_2 ($h_1 \geq h_2$)



- The supporting reinforcement is in addition to that required for other reasons
- The supporting links may be placed in a zone beyond the intersection of beams

Columns (2)

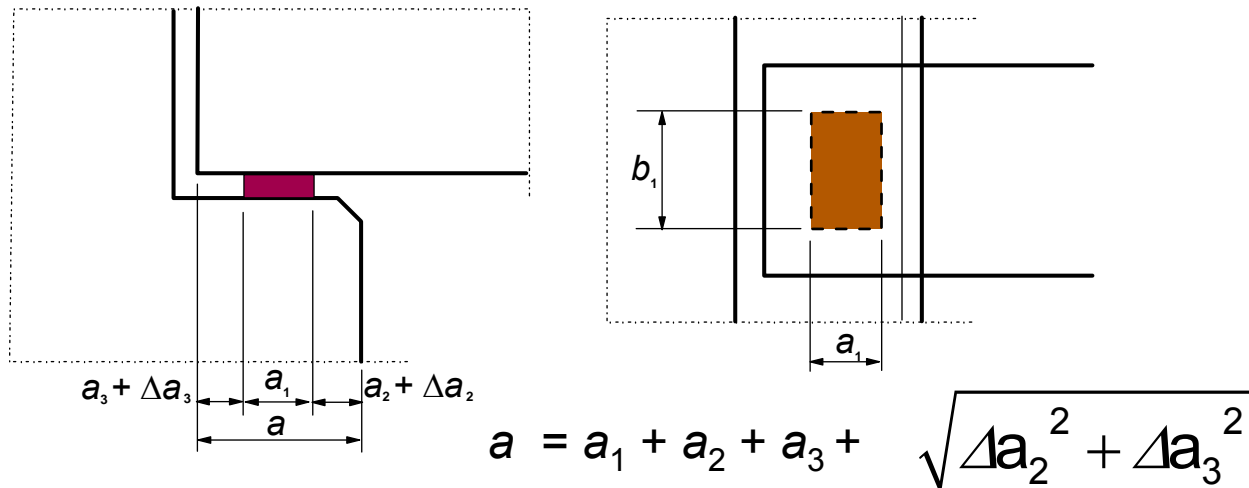
(9.5.3)



- $s_{cl,tmax} = 20 \times \phi_{min}; b; 400mm$
- $s_{cl,tmax}$ should be reduced by a factor 0,6:
 - in sections within h above or below a beam or slab
 - near lapped joints where $\phi > 14$. A minimum of 3 bars is rqd. in lap length

Additional rules for precast concrete

Bearing definitions (10.9.5)



a_1 net bearing length = $F_{Ed} / (b_1 f_{Rd})$, but \geq min. value

F_{Ed} design value of support reaction

b_1 net bearing width

f_{Rd} design value of bearing strength

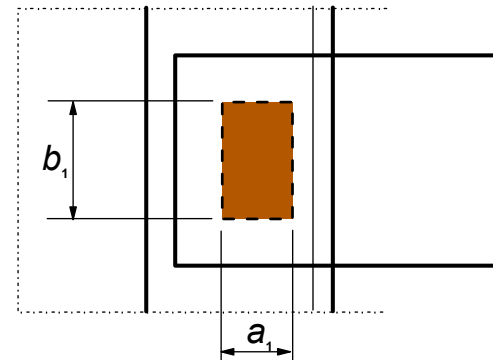
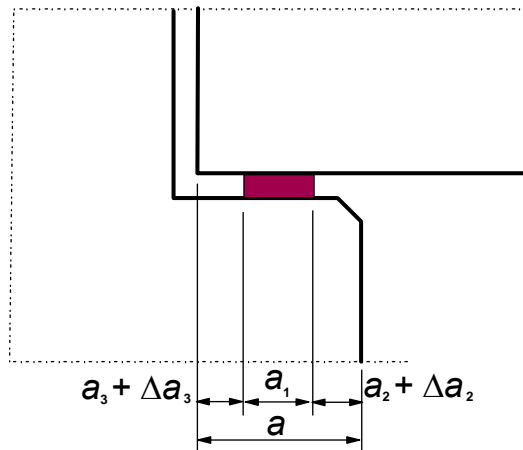
a_2 distance assumed ineffective beyond outer end of supporting member

a_3 similar distance for supported member

Δa_2 allowance for tolerances for the distance between supporting members

$\Delta a_3 = l_n / 2500$, l_n is length of member

Bearing definitions (10.9.5)

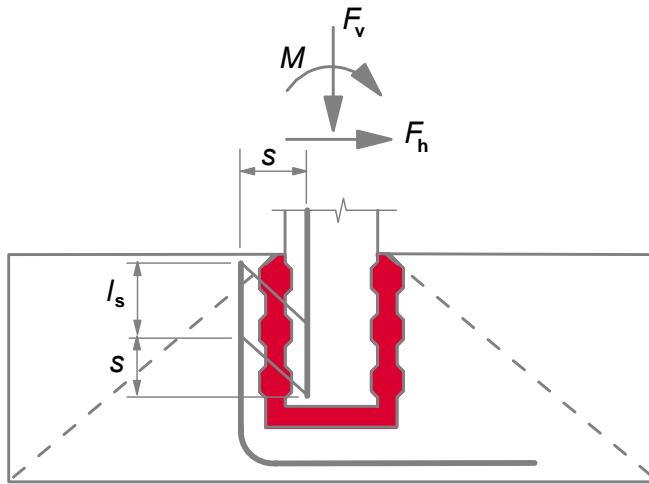


$$a = a_1 + a_2 + a_3 + \sqrt{\Delta a_2^2 + \Delta a_3^2}$$

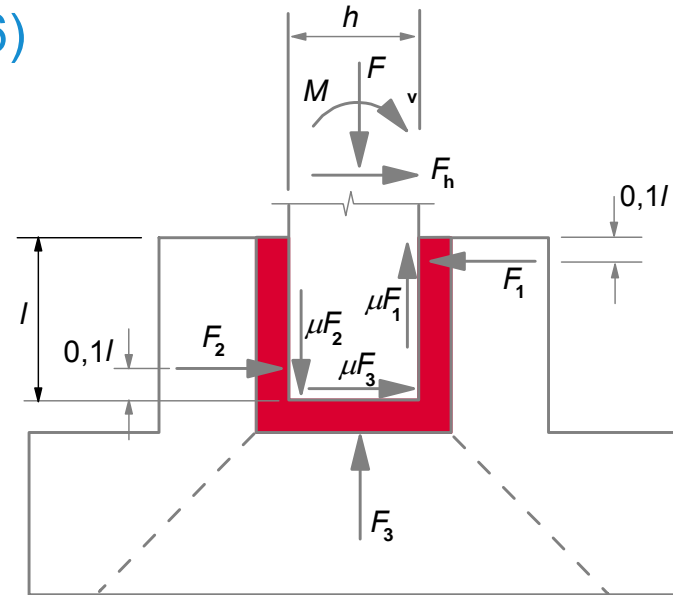
Minimum value of a_1 in mm

Relative bearing stress, σ_{Ed}/f_{cd}	$\leq 0,15$	0,15 - 0,4	$> 0,4$
Line supports (floors, roofs)	25	30	40
Ribbed floors and purlins	55	70	80
Concentrated supports (beams)	90	110	140

Pocket foundations (10.9.6)



$$l \leq s + l_s$$

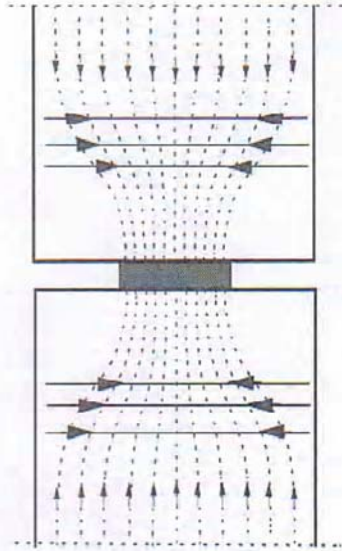


$$l \leq 1.2 h$$

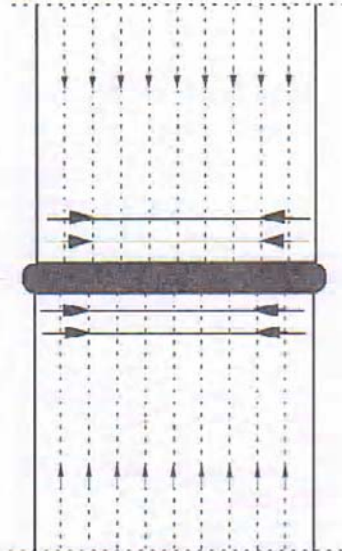
Special attention should be paid to:

- shear resistance of column ends
- detailing of reinforcement for F_1 in top of pocket walls
- punching resistance of the footing slab under the column force

Connections transmitting compressive forces



Concentrated bearing



Soft bearing

For soft bearings, in the absence of a more accurate analysis, the reinforcement may be taken as:

$$A_s = 0,25 (t/h) F_{ed}/f_{yd}$$

Where:

t = padding thickness

h = dimension of padding in direction of reinforcement

F_{ed} = design compressive force on connection

Lightweight aggregate concrete

Prof.dr.ir. J.C. Walraven

22 February 2008

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Lightweight concrete structures in the USA



Nappa bridge
California 1977



Oronado bridge San Diego



52 m prestressed concrete beams, Lafayette
USA

Rilem Standard test

Raftsundet Bridge, Norway



Antioch Bridge california

Qualification of lightweight aggregate concrete (LWAC)

Lightweight aggregate concrete is a concrete having a closed structure and an oven dry density of not more than 2200 kg/m^3 consisting of or containing a proportion of artificial or natural lightweight aggregates having a density of less than 2000 kg/m^3



Lightweight concrete density classification

Density classification

Density class		1,0	1,2	1,4	1,6	1,8	2,0
Oven dry density (kg/m ³)		801- 1000	1001- 1200	1201- 1400	1401- 1600	1601- 1800	1801- 2000
Density (kg/m ³)	Plain concrete	1050	1250	1450	1650	1850	2050
	Reinforced concrete	1150	1350	1550	1750	1950	2150

Conversion factors for mechanical properties

The material properties of lightweight concrete are related to the corresponding properties of normal concrete. The following conversion factors are used:

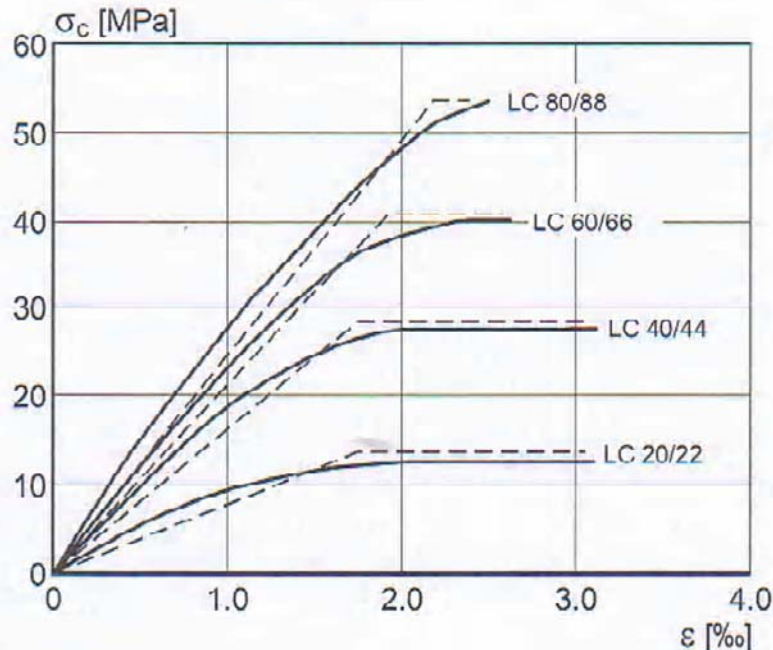
η_E	conversion factor for the calculation of the modulus of elasticity
η_1	coefficient for the determination of the tensile strength
η_2	coefficient for the determination of the creep coefficient
η_3	coefficient for the determination of the drying shrinkage
ρ	oven-dry density of lightweight aggregate concrete in kg/m ³



Antioch Bridge, California, 1977

Design stress strain relations for LWAC

The design stress strain relations for LWAC differ in two respects from those for NDC.

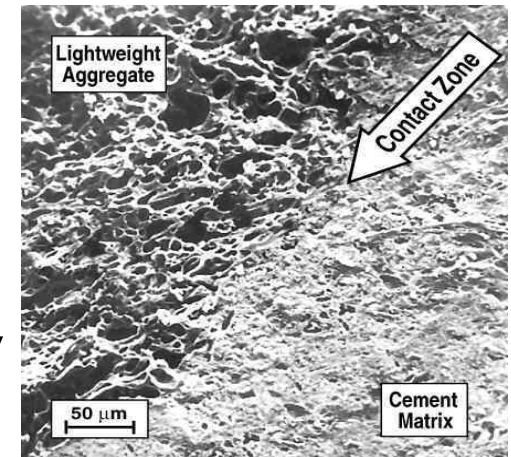


- The advisory value for the strength is lower than for NDC (sustained loading factor 0,85 in stead of 1,0)
- The ultimate strain $\varepsilon_{l, cu}$ is reduced with a factor $\eta_1 = 0,40 + 0,60\rho/2200$

Shrinkage of LWAC

The drying shrinkage values for lightweight concrete (concrete class \geq LC20/25) can be obtained by multiplying the values for normal density concrete for NDC with a factor $\eta_3=1,2$

The values for autogenous shrinkage of NDC represent a lower limit for those of LWAC, where no supply of water from the aggregate to the drying microstructure is possible. If water-saturated, or even partially saturated lightweight concrete is used, the autogenous shrinkage values will considerably be reduced (water stored in LWAC particles is extracted from aggregate particles into matrix, reducing the effect of self-dessication)

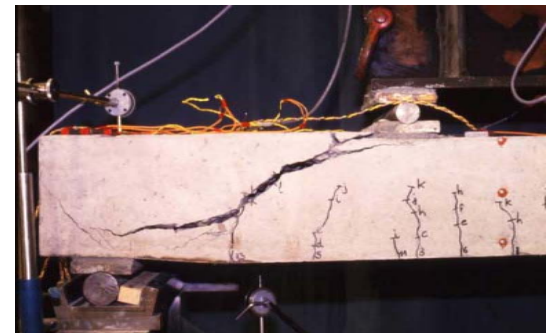


Shear capacity of LWAC members

The shear resistance of members without shear reinforcement is calculated by:

$$V_{lRd,ct} = \{ (0,15 / \gamma_c) \eta_1 k (100 \rho_l f_{lck})^{1/3} + 0,15 \sigma_{cp} \} b_w d$$

where the factor $\eta_1 = 0,40 + 0,60 \rho / 2200$ is the only difference with the relation for NDC



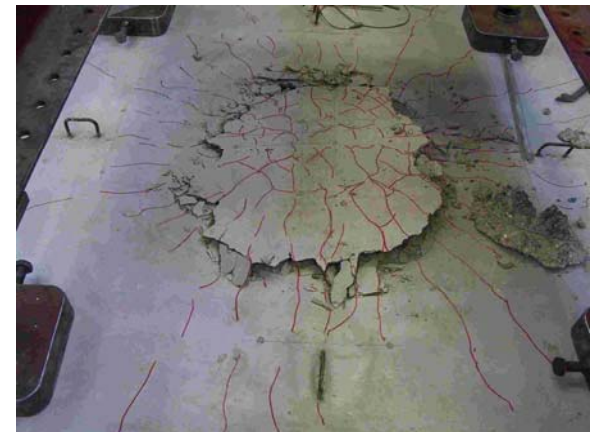
Punching shear resistance

Like in the case for shear of LWAC members, also the punching shear resistance of LWAC slab is obtained using the reduction factor $\eta_1 = 0,4 + 0,6\rho/2200$. the punching shear resistance of a lightweight concrete slab follows from:

$$V_{Rd,c} = (C_{lRd,c} k \eta_1 (100 \rho_l f_{lck})^{1/3} + 0,08 \sigma_{cp} \geq \eta_1 v_{lmin} + 0,08 \sigma_{cp}$$

where $C_{lRd,c} = 0,15/\gamma_c$

(in stead of the $0,18/\gamma_c$ for NDC)



Plain and lightly reinforced concrete

Prof.dr.ir. J.C. Walraven

22 February 2008

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Field of application

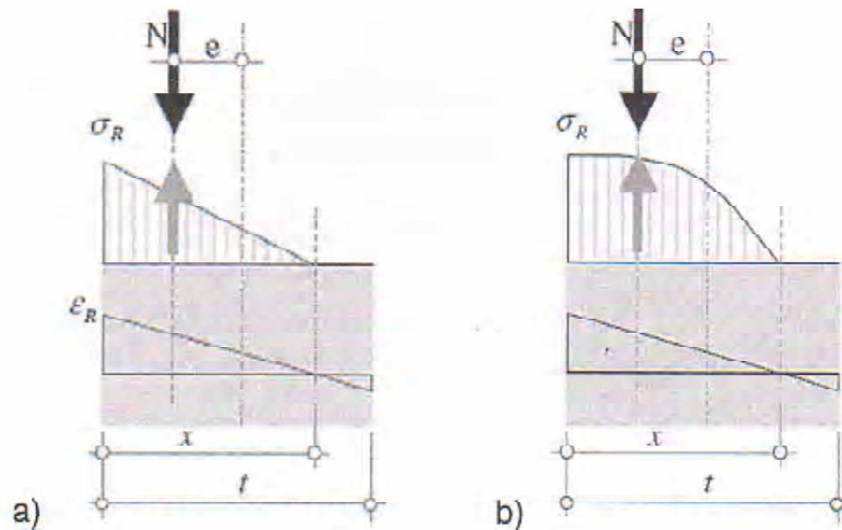
Members for which the effect of dynamic action may be ignored

- Members mainly subjected to compression other than due to prestressing, e.g. walls, columns, arches, vaults and tunnels
- Strip and pad footings for foundations
- Retaining walls
- Piles whose diameter is $\geq 600\text{mm}$ and where $N_{ed}/A_c \leq 0,3f_{ck}$



Additional design assumptions

12.3.1 Due to the less ductile properties of plain concrete, the design values should be reduced. The advisory reduction factor is 0,8



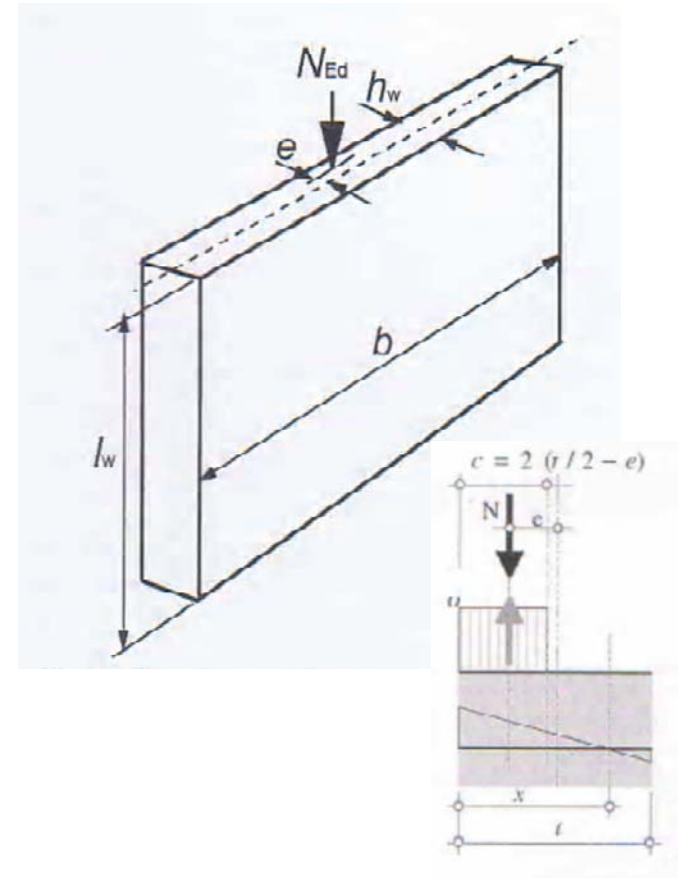
ULS: design resistance to bending and axial failure

The axial resistance N_{Rd} of a rectangular cross-section with a uniaxial eccentricity e , in the direction of h_w , may be taken as:

$$N_{Rd} = \eta f_{cd} b h (1 - 2e/h_w)$$

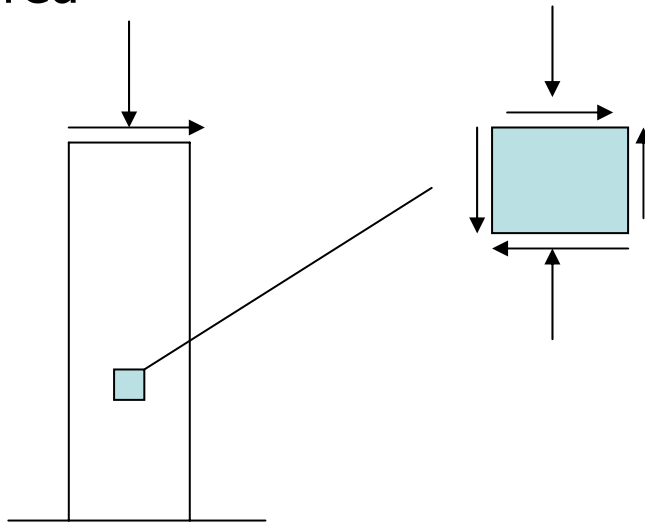
where

ηf_{cd} is the design compressive strength belonging to the block shaped stress-strain relation



Shear

12.6.3 (1): “In plain concrete members account may be taken of the concrete tensile strength in the ultimate limit state for shear, provided that either by calculation or by experience brittle failure can be excluded and adequate resistance can be ensured”



Using Mohr's circle it should be demonstrated that nowhere in the structure the principal concrete tensile stress of the concrete exceeds the design tensile strength f_{ctk}

Simplified design method for walls and columns

In the absence of a more rigorous approach, the design resistance in terms of axial force slender wall or column in plain concrete may be calculated as follows:

$$N_{Rd} = b \cdot h_w \cdot f_{cd} \cdot \phi$$

where

N_{Rd} is the axial resistance

b is the overall width of the cross-section

h_w is the overall depth of the cross-section

ϕ is a factor taking account eccentricity, including second order effects

$$\phi = 1,14 \cdot (1 - 2e_{tot}/h_w) - 0,02 l_0/h_w \leq (1 - 2e_{tot}/h_w)$$



Eurocodes: a big step forward:

- The rules within the Eurocode are very wide ranging (much better than all existing national codes)
- The fact that the Eurocodes cover a range of structural materials is an advantage to designers
- The use of common loading suggests a logical and economical approach to design
- The Eurocodes are written in a way that allows the designer to adopt the most modern design techniques
- The Eurocodes are unique among modern codes in that they allow for local variations in climate and custom, and can thus easily be adopted for safe and economic use

Eurocode: only for Europe?

